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A NEW DETERMINATION OF THE ELECTROCHEMICAL EQUIVALENT OF SILVER.

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THE discrepancy of the values for the mechanical equivalent of heat obtained by the electrical method and those found by the direct method[1] has suggested the possibility of the difference being due to an error in the standards used. The value of the ampère was suspected to be wrong and this led us to attempt a redetermination of its value by a new method. The electrochemical equivalent of silver has been repeatedly determined and was found to be 0.0011156 grm. by Mascart,[2] 0.00111794 grm. by Lord Rayleigh and Mrs. Sidgwick[3] and 0.0011183 grm. by Fr. and W. Kohlrausch.[4] In the first two cases an electrodynamic balance was used; in the last an accurate tangent galvanometer. Heydweiller[5] compared Lord Rayleigh's and Kolrausch's methods directly and came to the conclusion that the error in the two methods could not be larger than 0.1%. Köpsel[6] found 0.0011174 grm. with a balance built according to von Helmholtz's directions. The experi-

[1] A table with the results obtained by various methods since 1867 is given in Phil. Mag., 39, 160, 1895; see also Day, Phys. Rev., VI., 193, 1898, and Johns Hopkins Univ. Circ., XVII., 54, 1898.

[2] Journ. de Phys. (2) 1, 109, 1882, and 3, 83, 1884.

[3] Proc. Roy. Soc. 37, 144, 1884, or Phil. Trans. 2, 411, 1884.

[4] Sitzungsber. d. phys.-med. Ges., Würzburg, 1884, Wied. Ann., 27, 1, 1886.

[5] Inaug.-diss., Würzburg, 1886.

[6] Wied. Ann., 31, 268, 1887.

ments by Pellat and Potier,[1] using Pellat's absolute electrodynamometer, gave 0.0011192 grm. For a number of years Kahle has been at work at the Imperial Institute in Berlin using Helmholtz's current-balance,[2] but the definite report of this valuable investigation has not yet been published. In the preliminary report 0.0011182 grm. and 0.0011193 grm. are given according as the silver was deposited from a fresh or old solution.[3]

In all the experiments mentioned the value of the electrochemical equivalent of silver depends upon the accurate knowledge either of the acceleration due to gravity or of the horizontal intensity of the earth's magnetic field. We proposed to measure its value entirely independent of g or H by balancing the moment due to the electrodynamic action of the two coils of an absolute electrodynamometer, designed by us and made under our direction, against the torsional moment of a wire with very small elastic fatigue. The measurements are finally reduced to measurements of mass and length only and we believe our result to be accurate to at least one in five thousand.

Description of the Method. In our determination we made use of Poggendorff's potentiometer method for comparing electromotive forces. First, we found the ratio between the E.M.F. of two Carhart-Clark cells immersed in a large water-bath and the potential difference at the terminals of a coil of magnanin wire, when a current flowed through it, which just balanced the torsional moment of the suspending wire of the electrodynamometer due to one complete twist of the torsion head. The absolute value of this current was calculated from the dimensions of the instrument.

Let I be the calculated current,

E the E. M. F. of the cells,

R_1 the resistance necessary to balance E,

R_2 the resistance necessary to balance the P. D. due to I,

R the resistance of the manganin coil;

then
$$I = \frac{R_2}{R_1}\frac{E}{R}. \qquad (1)$$

[1] Journ. de Phys., (2) 9, 381, 1890.

[2] Wied. Ann., 59, 532, 1896.

[3] Zeitschr. f. Instrum. Kunde, 17, 144, 1897, and 18, 141, 1898.

Secondly, we balanced the E. M. F. of the cells against the Pot. Diff. due to the silver depositing current.

Let I' be this current,

R_1' and R_2' the resistances necessary for the balances,

E and R the same as before,

then
$$I' = \frac{R_2'}{R_1'} \frac{E}{R}. \tag{2}$$

The silver deposited by the current I' is

$$q = zI't \text{ grms.} \tag{3}$$

where z is the electrochemical equivalent of silver ;

but
$$I' = I \frac{R_1}{R_2} \frac{R_2'}{R'}, \tag{4}$$

therefore
$$z = \frac{q}{It} \frac{R_1'}{R_1} \frac{R_2}{R_2'}. \tag{5}$$

We are thus independent of the absolute value of the E. M. F. of the standard cells and the resistance of the manganin coil.

We chose Carhart-Clark cells, in which the zinc sulphate solution is free from crystals at ordinary temperatures, rather than normal Clark cells, in which the zinc sulphate solution is always in presence of crystals, to avoid all time lag of electromotive force due to changing density after change of temperature. The Carhart-Clark cells have the further advantage of having only about one-half as great a temperature coefficient as the normal Clark cells.

The manganin coil had a negligible change in resistance within the limits of the temperatures at which we used it. The current through the silver voltameters was not measured directly by the electrodynamometer. The reason for not doing so was that the electrodynamometer was extremely sensitive, and it would have been almost, if not quite, impossible to control the current through the circuit from the beginning to the end of a set of observations at such a value as to balance the turning moment of the suspension due to exactly one turn of the torsion head of the instrument. This difficulty was due to several causes: first, the current diminished

gradually, due to polarization of the battery and heating of the coils; second, slight changes of temperature produced changes in the wire suspending the movable coil, from which resulted changes in the torsional constant and in the zero point; third, there were gradual though very small changes in the zero point due to elastic fatigue of the suspension. Besides these reasons it should be mentioned that an absolute electrodynamometer should be used with all four possible permutations of direct and reversed current through both coils.

Poggendorff's method gave an almost ideal means of holding all these diverse disturbing factors in control. Fig. 1 shows the arrangement of our apparatus.

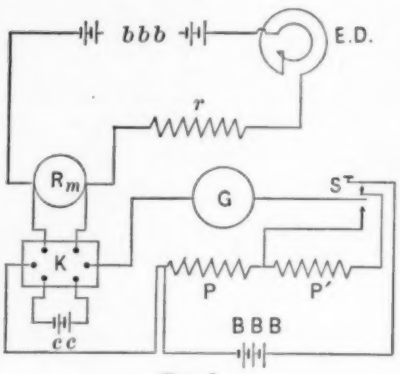

Fig. 1.

In our experiments we used either two or three storage cells *BBB* to produce the current through two carefully adjusted resistance boxes *P*, *P'* by Nalder Brothers. The total resistance was kept at 11,110 ohms, and the fall of potential through a portion *P* of this resistance, was balanced against the electromotive force of the standard cells *cc* or against the fall of potential through the manganin coil R_m. *K* is a Pohl's commutator, *G* a sensitive Kelvin galvanometer and *S* a successive contact key. In one arrangement the manganin coil was in circuit with the storage battery *bbb* and the electrodynamometer *ED*, in the other two silver voltameters were included in the circuit also.

The silver nitrate solution was made by weight 15% c. p. silver nitrate and 85% distilled water with a trace of silver oxide added to insure a neutral solution. The same solution was used on successive days. Kahle, in a recent paper,[1] has called attention to a difference he finds in the electrochemical equivalent of silver when determined from "fresh" and "used" solutions. We did not prepare a solution afresh for each determination. The close agreement

[1] Zeitschr. f. Instr., 17, 144, 1897.

between our different results would indicate that our solution remained constant for at least the last four days on which the experiments, which were used for the final calculations, were made. We used for silver voltameters two platinum bowls of 7.5 cm. diameter and 2.5 cm. height as kathodes and pure silver plates about 2.5 cm. diameter wrapped in filter paper and suspended by platinum wires near the centers of the bowls as anodes. We used currents slightly less than one ampère and kept the circuit closed for about one hour. Before using the bowls they were carefully cleaned, dried and weighed with accurate weights to the nearest tenth of a milligram by means of the method of double weighing. After opening the circuit the anode was removed, the solution poured back into its bottle and the bowls carefully washed twice with distilled water and left to soak for a day, during which time the water was renewed once. The bowls were emptied, dried in an air bath at about 130° and left to cool in desiccators. They were then weighed as before.

The Electrodynamometer.[1]

Our electrodynamometer consisted of a fixed coil of 576 turns in a single layer of 41.633 cm. mean length and 48.118 cm. mean diameter and a movable coil of 45 turns of 8.805 cm. mean length and 10.0187 cm. mean diameter suspended at the center of the fixed coil by a single phosphor-bronze wire 0.0355 cm. in diameter and 100.5 cm. long. The current was led into the movable coil through two mercury cups below and in line with the suspension. The torque due to the current in the fixed coil and the conductors between the movable coil and the mercury cups was avoided by using a symmetrically divided circuit leading around the upper mercury cup to the mercury in the lower cup. There was, however, a slight torque due to the action of the current in the conductor leading from the upper cup to the point beyond which the lead wires were twisted together. The lead wires from the terminals of the fixed coil were carried along the surface, parallel to the axis, to binding posts at diagonally opposite corners of a square and the lead wires from the suspended coil were brought out to the other corners of

[1] The instrument was constructed and many of the minor details suggested by Mr. R. H. Miller, the instrument maker to the university.

this square. The action of these lead wires is shown in the difference between the values of the current needed to balance a right-handed or left-handed twist of the suspension. The error due to this asymmetry was eliminated by reversing the current in the fixed coil.

In order to insure greatest accuracy in determining the angle of torsion we used one complete turn of the torsion head, which was determined by means of a mirror on the torsion head and a telescope and scale at a distance of about 150 cm. The torsion head could be read by means of a vernier to one ten-thousandth of a circumference, but we did not in general make readings on it directly, as the telescope, mirror and scale made a much more accurate determination of a complete turn possible. In our preliminary experiments we made use of half and quarter turns also and obtained concordant results.

The cylinder holding the fixed coil was built up of dry, clear pine wood about 2.5 cm. thick. Each layer consisted of five pieces, and successive layers broke joints. The grain of the wood ran around the cylinder. By this arrangement we succeeded in keeping the cylinder of uniform diameter, although the wood shrunk a little (one in five thousand) after it was put together. The layers were fastened together by both glue and brass screws. After the cylinder was built up, it was turned in a large lathe to the required diameter. It was then given many coats of shellac, outside and in, to keep it as nearly as possible moisture proof. It was then wound with 576 turns of number 23 (B. & S. gauge) single cotton covered wire, which afterwards was given eight coats of shellac to ensure good insulation. The insulation was tested and found perfect. The winding was helical except near the small opening through which the suspension passed, where short portions of six turns were displaced. To make the effect of this displacement as small as possible, three wires on each side of the hole were half sunk into the wood of the cylinder and three others were wound on top of them. By this means the mean radius of the coil was unchanged and the effect on the mean field within the suspended coil was not changed materially. The suspended coil was wound on a hollow vulcanite cylinder. The conductor was a copper ribbon made by rolling a

number 25 (B. & S. gauge) wire down to a thickness of .024 cm. The cylinder was connected to the suspension by means of a small brass rod about 0.2 cm. in diameter which was soldered above into a hollow brass cylinder, and below into a brass plate which extended the length of the cylinder and was fastened by screws to its ends. Where the brass rod passed from above into the vulcanite it was cut away on two sides so as to allow the coil to keep in a nearly helical form. The suspending wire was soldered into a brass cylinder which was fitted and pinned into the hollow brass cylinder mentioned above. The terminals of the suspended coil were at points along the bottom element of the vulcanite cylinder and were connected along the inner surface, one to the upper mercury cup by a straight wire soldered to a small brass pin with a platinum wire tip in line with the suspension, the other by a Y-shaped connection to two symmetrical rods which passed out on opposite sides of the upper mercury cup and were soldered into a cross bar which held a brass rod in line with the suspension. This rod supported a mirror and an adjusting weight, and held at its end a platinum wire which projected into the lower mercury cup. The adjusting weight was designed to make the mass of the system exactly equal to the mass whose moment of inertia and period of torsional vibration determined the torsional constant of the suspending wire. (See Fig. 3.)

The dimensions of both the suspended and fixed coils were intended to fulfill the requirement laid down by Gray[1] that the length and radius should be in the proportion of $\sqrt{3} : 1$. When both coils fulfill this requirement and when their centers coincide, all the terms of the series expressing the torque between the coils disappear except the first, seventh and following odd terms. If the dimensions of the fixed coil are large in comparison with those of the movable coil, all terms after the first may be neglected. Owing to shrinkage of the wood of the fixed cylinder, the fixed coil became somewhat too short to fulfill Gray's requirements; the suspended coil was much closer. Nevertheless, the correction necessary does not amount to even one in one hundred thousand.

We carefully leveled the fixed coil by means of a long level.

[1] Absolute Measurements in Electricity and Magnetism, Vol. II., p. 277, or Phil. Mag., 33, p. 62.

Then we adjusted the zero position of the movable coil by means of a set of concentric squares and circles on a circular board which was fitted into one end of the cylinder of the fixed coil. The suspended coil was turned parallel to the fixed coil and the height of the torsion head was adjusted until the center of the movable coil was opposite the center of the squares and circles. The torsion head was then given a half turn to test the level of the movable coil. This was found perfect. Next a scale was placed across the other end of the fixed coil and the leveling screws under one side of the instrument were adjusted to bring the axis of the fixed coil and the suspension into the same plane. The adjusting screws at the torsion head were then used to bring the line of the suspension into the middle circle of the large cylinder which corresponded very closely with the middle of the fixed coil. The positions of the murcury cups were then adjusted to bring the platinum tips to their centers. All the tests were then repeated and the position of the movable coil appeared exactly central and horizontal. We then turned the torsion head one-quarter revolution and verified the position by means of the concentric squares and the flat ends of the movable cylinder. Any error in the level of either coil would introduce an error proportional to the difference between unity and the square root of the cosine of the angle. No error of the magnitude of one part in ten thousand could have been present.

The electro-dynamometer was set up with the axis of the fixed coil approximately in the magnetic meridian and the axis of the movable coil exactly at right angles to that of the fixed coil. We chose this position to avoid having the movable coil act like a dip needle when the current was on. The effect of the earth's field was considerable, but was eliminated by reversing the current in both coils and adjusting its value until the same torque was produced. Let the current through the coils be represented by i_1 and $-i_2$ in the two cases and let a and b be coefficients to take account of the dimensions of the instrument and the value of the earth's field, then the torque f required to hold the movable coil in its zero position may be represented in the two cases as follows:

$$f = ai_1^2 + bi_1, \tag{6}$$

and $$f = ai_2^2 - bi_2; \tag{7}$$

eliminate b and we find $$f = ai_1i_2, \tag{8}$$

which shows that excluding the earth's field a current whose value was a mean proportional between the numerical values of the two actual currents, would had produced the same torque as each did with the earth's field acting.

To eliminate the action of the lead wires we must consider that a is replaced in one case by $c + d$ and with the connections of the coils changed by $c - d$, which gives two equations for the same torque f. i_3 and $-i_4$ are the new currents corresponding to i_1 and i_2.

$$f = (c + d)\, i_1i_2. \tag{9}$$

$$f = (c - d)\, i_3i_4. \tag{10}$$

$$f = 2c\left(\frac{i_1i_2i_3i_4}{i_1i_2 + i_3i_4}\right) = c\sqrt{i_1i_2i_3i_4} \text{ nearly.} \tag{11}$$

This approximation, which is more convenient than the exact expression, introduces no error larger than one part in a million in any of our computations.

The Suspending Wire.

The suspension of the movable coil was a phosphor-bronze wire soldered into brass cylinders at both ends. The larger cylinder served to hold the upper end of the wire in the torsion head; the smaller could be coupled to the movable coil in the manner described above. These brass cylinders with the position of the wire reversed served, one to support a hollow brass cylinder of known moment of inertia used in determining the torsional coefficient of the wire, the other to support the wire itself.

The wire obeyed Hook's law almost exactly, as will appear from observations made while the wire was undergoing torsional vibrations. The period of vibration in vacuo was affected by a change in amplitude from a small arc to a complete rotation by about one part in twelve thousand; and the logarithmic decrement was extremely small and practically the same at all amplitudes. A rise of temperature increased the period by about one part in five thousand per degree centigrade.

Elastic fatigue produced a small effect on the wire. If the wire, when supporting the movable coil, was given one complete twist and held in this position for several minutes and then allowed to untwist, it was noticed that the position of equilibrium was displaced in the direction of the twist by about one one-thousandth of a complete turn. Nearly the full effect was reached after ten minutes' twisting. On untwisting, it took a much longer time for the residual twist to disappear. In the use of the electrodynamometer we eliminated the effect of the elastic fatigue by twisting the wire once round and leaving it twisted for several minutes before beginning our observations. The current was adjusted until the movable coil hung in its zero position at right angles to the fixed coil. The circuit was then opened and the torsion head turned back to its zero position. The position of equilibrium of the movable coil was then quickly determined by using an auxiliary battery, and the torsion head was again given a full turn and enough more to allow for the deviation of the position of equilibrium from the zero position. The circuit was then closed and the current adjusted until the movable coil was in its zero position again. The circuit was then broken and the torsion head turned back one complete revolution and the position of equilibrium redetermined. This was repeated until the position of equilibrium was the zero position, or until successive positions lay on both sides of zero. The strength of current was indicated by the number of ohms in the box P of Fig 1.

The question naturally arises: Did our method of allowing for the slight elastic fatigue introduce any error? We think not. We made a number of observations to test this. We allowed the suspension of the electrodynamometer to rest over night. We then completed the circuit but did not twist the suspension. We adjusted the current to the right value to correspond to a complete tur n andallowed the current to become quite steady without twisting the wire. After about twenty minutes we opened the circuit, noted the position of equilibrium which in general changed slightly owing to a change in temperature and closed the circuit again. After adjusting the zero for the change of temperature we then turned the torsion head a complete revolution and determined the number of ohms in P (Fig. 1) required to balance the current and noted the temperatures. We

kept adjusting the current so as to keep the movable coil in its zero position and noted the corresponding number of ohms in *P*. We continued this over a period of half an hour, or until conditions had become constant. From these observations we estimated by extrapolation what number of ohms in *P* would have corresponded to a balance at the instant of twisting the suspension. We then opened the circuit, adjusted the zero and determined the ohms corresponding to one complete turn of the suspension from this last zero. This value remained constant except for change of temperature, showing that the change due to elastic fatigue had ceased. Making allowance for slight changes in temperature we found that, *as nearly as we could determine, the current required to balance corresponded exactly to the current which we estimated would have produced a balance at the instant of turning the torsion head.* The difference was certainly not as great as one part in ten thousand. After several tests of this kind we got rid of the elastic fatigue before beginning observations. If we had neglected the effect of elastic fatigue, we would have got a value for the square of the current too large by one part in one thousand. This corresponds to an error of one part in two thousand for the current itself.

The temperature of the suspending wire was estimated at the time of each balance by means of two thermometers, one inside the case and one against the brass tube surrounding the wire. (The second thermometer was that used when the torsional constant of the wire was determined.) To facilitate rapidity of work the motion of the movable coil was limited to a small arc by the prongs of a wooden fork (see Fig. 3). Before the fork was used, we found that the suspended coil would, on occasion, swing through the position of stable equilibrium, on through a position of unstable equilibrium and finally come to rest in another position of stable equilibrium. The torque in the various positions may be represented by the combination of a simple cosine curve with an inclined straight line. The former represents the torque of the electric forces, the latter that of the suspending wire. In combining the two torques, account must be taken of the angle through which the torsion head has been turned. For equilibrium positions must be found in which the two ordinates are numerically equal but opposite in sign. The com-

bined curve has positions of equilibrium wherever the ordinate becomes zero. The positive values of this curve tend to increase the angle, the negative to decrease it. The odd positions correspond to stable and the even to unstable equilibrium. We used the second

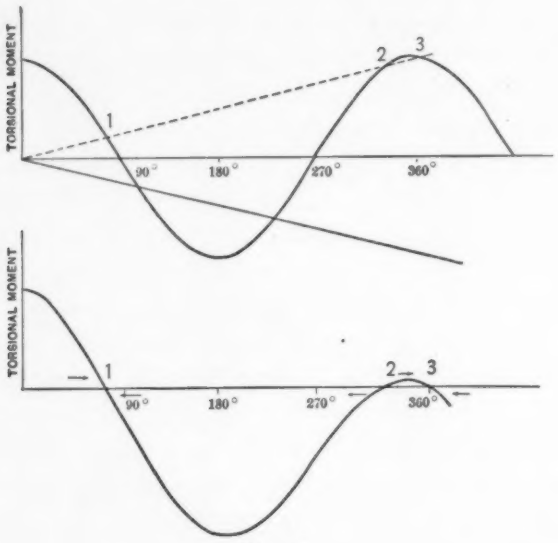

Fig. 4.

position of stable equilibrium, as shown at 3, and adjusted the current until it corresponded to one complete turn of the torsion head with the movable coil in its zero position. With this arrangement we found that a position of unstable equilibrium was quite near by, and until we had used the wooden fork to stop the coil, it was quite difficult to avoid having the coil turn around while adjusting the current to its right value. The wooden fork completely eliminated this trouble.

TORSIONAL COEFFICIENT OF THE SUSPENDING WIRE.

The torsional coefficient of the phosphor-bronze suspension was investigated very carefully. We avoided changes in the length of the wire used by soldering its ends into the cylinders mentioned before. This was done at as low a temperature as possible. The larger cylinder fitted as exactly as possible into a hollow cylinder.

Two of these hollow cylinders were used by us and were made from rolled brass bars which we thought more uniform than cast brass. The moment of inertia of the combination was determined very carefully. We weighed them with brass weights which we compared with a set of standard weights, tested by the United States Bureau of Weights and Measures and we measured their dimensions by means of a Brown and Sharpe vernier caliper. We tested the graduation of the vernier caliper by means of a dividing engine and found it accurate. As the dimensions of the movable coil were measured by the same instrument any error due to a faulty unit would disappear. The length and outer radius of the hollow cylinders were in the ratio of $\sqrt{3} : 1$ [1] and consequently any slight error in the axis of rotation would produce a minimum disturbing effect on the moment of inertia. To the bottom of the central cylinder a small mirror was fastened with glue. The moment of inertia of both mirror and glue was computed and added to that of the brass. The moment of inertia of the wire was negligibly small. The second hollow brass cylinder—as nearly as convenient the duplicate of the first—was provided as a check to the first. It was thought that if air holes were present in either cylinder or in both, it would be hardly probable that the same results would be obtained from both. Identical values from the torsional constant of the wire were obtained from both, and it is probable therefore that the real and calculated moments of inertia were the same. Cylinder *I* was taken as our standard and cylinder *II* was used as a check only.

The inner smaller cylinder had at its one end a short projection, on which the larger hollow cylinders rested. Its moment of inertia is, therefore, considering it as two cylinders of diameters d and D, and length L and l, respectively,

$$I = \frac{M}{8}\left(\frac{La^4 + lD^4}{La^2 + lD^2}\right). \qquad (12)$$

[1] Limb, Compt. rend., 114, 1057, 1892.

DATA FOR THE BRASS CYLINDERS.

A. Inner Cylinder.

$M = 47.547$ grms.
$d = 1.268$ cms.
$D = 1.378$ cms.
$L = 4.331$ cms.
$l = 0.085$ cms.
$I = 9.5599$ cm.^{2} grm.

B. Hollow Cylinder *I.*

$M = 675.950$ grms.
$D = 4.9915$ cms.
$d = 1.272$ cms.
$L = 4.3345$ cms.
$I = 2241.87$ cm.^{2} grm.

D. Moment of Inertia of Mirror.

$I = 0.465$

C. Hollow Cylinder *II.*

$M = 674.3474$ grms.
$D = 4.9915$ cms.
$d = 1.272$ cms.
$L = 4.324$ cms.
$I = 2236.56$ cm.^{2} grm.

Total Moment of Inertia for Cylinder *I* $= 2251.89$ cm.^{2} grm.
for Cylinder *II* $= 2246.58$ cm.^{2} grm.

We made a long series of observations on the periods of torsional vibration of the system. To avoid the disturbing influence of the air, we enclosed the system in a long glass vacuum cylinder set in bearings at both ends. The system was set in torsional vibrations by turning the glass cylinder. Within the cylinder was a good thermometer reading directly to 0°.2 C. and a small barometer. During tests the pressure of the air was kept down to about 2.5 cm. of mercury, although a pressure of several times that amount did not affect the period of vibration by more than a negligible amount (for our purposes). We noticed that when the tension of the wire was relieved for some time, a considerable time was necessary before renewed tension brought the wire to the same condition as before. This effect was very slight and produced a variation in the period of vibration of the order of less than one part in ten thousand.

We carefully investigated the temperature coefficient of the wire, and found in general that the period increased with the temperature at the rate of one part in five thousand per degree centigrade. From observations with rapidly falling and rising temperatures near zero, we obtained some evidence of a hysteresis effect in the period

of vibration. Cold weather did not continue long enough for us to settle the matter definitely. With slowly rising or falling temperatures no such effect was detected.

We also carefully investigated the effect of amplitude of vibration on the period. We here noticed a small effect. The variation of the period between very small amplitudes of vibration and one whole turn from the position of equilibrium was of the order of one part in twelve thousand. We decided that this was due to friction with the small amount of air in the glass cylinder and molecular friction in the wire itself, and that, as these elements would be lacking when in use in the electrodynamometer, the right period for our purposes was the period for small amplitudes. The logarithmic decrement was very small and produced no appreciable effect on the period for small amplitudes.

From these investigations we concluded that the phosphor-bronze wire was well suited to use as a suspension in an absolute electrodynamometer. We were well satisfied with its action when in the instrument.

Determination of the Period.

Our method was a slight modification of Kohlrausch's method. On the bottom of the rotating cylinder was a mirror. Around the glass cylinder containing the rotating system was a circular scale subdivided into centimeters and millimeters at a distance of 15 cm. from the mirror.

The scale was observed through a telescope at a distance of about 150 cm. The instrument was set up in a room in which there was a Standard Riefler clock, the ticks of which could be heard distinctly. We determined the exact time, when a tick coincided with the passage of the mirror through the middle point of the swing and repeated these observations. The number of vibrations made during the interval between the first and the following coincidence was found by dividing the interval by an approximate period (obtained by inspection) and taking the whole number nearest the quotient. Dividing the interval by this whole number we obtain a more nearly accurate value for the period. Repeating this we can easily obtain an accuracy of 0.001 second within 200

swings. The method of procedure will become clear from the examples given in the following tables.

The absolute rate of the clock will not come into account in the final detemination of the electrochemical equivalent, if the time during which the silver is deposited is also measured with the same clock. We did not, therefore, determine the absolute rate, but referred all measurements of time made during the investigation to this Riefler clock as a standard.

Our first object was to determine in how far the elastic fatigue came into account in our measurements. Tables I. to III. give three series undertaken at different temperatures. In every case the logarithmic decrement is very small and nearly constant, from which we can conclude, that if there is any elastic fatigue, it is very slight. The correction for damping was too small to have any influence on our result. The amplitude of the vibrations varied considerably in the different tests and decreased to about one-half of its initial value during each series.

TABLE I.

Time of Transit.	No. of Swings.	Interval.	Period.	Right.	Left.	Amplit.	Log. dec.[1]	Temp.
37′ 39″	0			70.9	29.1	41.8		22°.6
47 01	47	562	11.957	70.2	29.7	40.5	.000292	22 .4
56 59	97	1160	11.958	69.5	30.3	39.2	.000288	22 .2
81 18	219	2619	11.9589	68.0	31.8	36.2	.000285	21 .9
130 20	465	5561	11.95912	65.25	34.44	30.81	.000285	22 .0
203 17	831	9938	11.95909	61.9	37.65	24.25	.000283	22 .0
232 59	980	11720	11.95918	60.94	39.0	21.94	.000286	22 .1

TABLE II.

Time of Transit.	No. of Swings.	Interval.	Period.	Right.	Left.	Amplit.	Log. dec.[1]	Temp.
38′ 15″	0			62.9	37.4	25.5		16°.0
61 10	115	1375	11.9570	———	———	———		15 .8
76 31	192	2296	11.9580	61.3	38.7	22.6	.000273	16 .0
94 51	284	3396	11.9570	60.75	39.3	21.45	.000264	16 .0
107 48	349	4173	11.9570	60.3	39.7	20.6	.000266	15 .5
202 48	825	9864	11.9564	57.8	42.2	15.6	.000259	15 .5
211 37	870	10402	11.8563	57.5	42.5	15.0	.000265	15 .7
229 21	959	11466	11.9561	57.12	42.88	14.24	.000264	15 .7
247 41	1051	12566	11.9562	56.75	43.22	13.53	.000262	15 .4

[1] In common logarithms per swing.

TABLE III.

Time of Transit.	No. of Swings.	Interval.	Period.	Right.	Left.	Amplit.	Log. dec.[1]	Temp.
42′ 07″	0	0		75.7	24.3	51.4		
47 41	28			75.35	24.55	50.8		7°.00
55 26	67			74.9	25.0	49.9	.000291	6 .65
58 01	80	954	11.925	74.8	25.15	49.65	.000188	6 .65
60 24	92	1097	11.9239	74.52	25.38	49.14	.000212	6 .70
62 59	105	1252	11.9238	———	———	———	———	6 .70
65 34	118	1407	11.9237	74.2	25.8	48.4	.000221	6 .70
70 32	143	1705	11.9237	73.9	26.0	47.9	.000214	6 .65

The second set was taken with the brass cylinder *I*, the first and third with the brass cylinder *II*.

It has been mentioned above that the amplitude of the vibrations had a small influence upon the period. The following series of observations shows this very well.

TABLE IV.

Time of Transit.	No. of Swings.	Interval.	Period.	Temperature.
7′55″	0	0		18°.30
23 27	78	932	11.94872	18 .25
31 37	119	1422	11.94958	18 .25
37 11	157	1876	11.94905	18 .20
62 41	275	3282	11.94907	18 .10
70 27	314	3752	11.94905	18 .12

The amplitude varied from 55 to 46.5 scale parts.

TABLE V.

Time of Transit.	No. of Swings.	Interval.	Period.	Temperature.
20′00″	0	0		18°.40
35 20	77	920	11.9480	18 .42
43 18	117	1398	11.9487	18 .20
50 52	155	1852	11.9484	18 .20
58 50	195	2330	11.9487	18 .15
66 36	234	2796	11.9487	18 .05
73 58	271	3238	11.9483	18 .05
81 56	311	3716	11.9485	18 .05

[1] In common logarithms per swing.

Amplitude varied from 9.5 to 8.8 scale divisions.

The difference between the period for the two amplitudes is even a little smaller than appears from this on account of the lower average temperature in the second case, which tends to decrease the period.

INFLUENCE OF TEMPERATURE UPON THE PERIOD.

While in the preceding experiments we had simply decided that the phosphor-bronze wire was well fitted for use in the electrodynamometer and no account had been taken of the thermometer correction, the following experiments were undertaken to determine the temperature coefficient of the wire and a new thermometer, subdivided into .2° C. was put into the vacuum cylinder.

The following table shows the effect of a change of temperature on the time of vibration of the system. The period was determined in the manner described above. We note here only the final period. These observations are selected out of a large number made with the same wire. The values in the second column denote the average temperature.

TABLE VI.

Date.	Temperature.	Period.
April 4th	14°.84	11.94083
5th	5 .91	11.92275
5th	9 .15	11.9283
9th	18 .20	11.94905
13th	19 .32	11.9527

We cannot be perfectly sure about the low temperatures, because it was rather difficult to keep the room at a constant temperature, while we were working in it. Nevertheless it seems apparent that the temperature-coefficient of the wire increases with rising temperature.

The observations in Table VI. were taken with the big hollow cylinder *II*, and before the wire was fixed for the final determinations. That the temperature coefficient was not changed when the wire was soldered into the brass cylinder, which was to hold the movable coil of the electrodynamometer, is apparent from the fol-

lowing table. These observations were taken with the wire, when ready for use in the electrodynamometer and with hollow cylinder *I*.

TABLE VII.

Date.	Temperature.	Period.
April 20th	20°.19	11.96791
June 30th	23 .06	11.97492
June 30th	24 .72	11.9798
June 31st	26 .89	11.9851

The first observation was taken before, the last three after, the determinations with the electrodynamometer.

The temperature coefficient between 15° and 30° is .00022.

The results are plotted in Fig. 5, *I* being the curve for Table VI., and *II* the curve for Table VII.

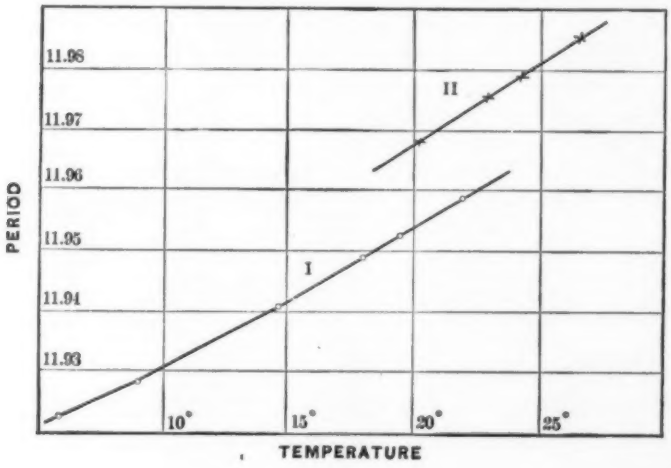

Fig. 5.

EXPERIMENTS WITH THE ELECTRODYNAMOMETER.

In our final experiments we balanced the moment of torsion due to a complete twist of the wire, against the electrodynamic action between the two coils of the electrodynamometer. The moment of torsion of the wire for a twist of unit angle is

$$w = \frac{4\pi^2 K}{T^2}, \tag{13}$$

where K denotes the moment of inertia of the rotating system and T the period of vibration corrected for damping.

The strength of the field at the center of the stationary coil is

$$H = \frac{4\pi N}{\sqrt{D^2 + L^2}} I = hI, \tag{14}$$

where N is the number of turns in the stationary coil,
D and L its mean diameter and mean length,
I the current expressed in C. G. S. units.

The area of movable coil, multiplied by the number of turns of the same is

$$A = \frac{\pi d^2 n}{4}; \tag{15}$$

so that the moment of torsion due to the action of the two coils upon each other is, if they are at right angles to each other,

$$w' = hAI^2 = \frac{\pi^2 N n d^2}{\sqrt{D^2 + L^2}} I^2. \tag{16}$$

The equation of equilibrium is then, the twist being 2π

$$\frac{\pi^2 N n d^2}{\sqrt{D^2 + L^2}} I^2 = 2\pi w = \frac{8\pi^3 K}{T^2}, \tag{17}$$

or

$$I = \frac{1}{Tr}\sqrt{\frac{2\pi K \sqrt{D^2 + L^2}}{Nn}}, \tag{18}$$

where r is the radius of the movable coil.

Substituting the numerical values given on pages 261 and 270, and taking for the time of vibration of our system 11.9853 at 27°, we obtain as the theoretical value of the *current necessary to balance the torsional moment due to a twist of 2π, of the wire at the temperature 27°*,

$$I = 0.98158 \text{ AMPÈRE.}$$

To compare this value with the silver depositing current, we made use of Poggendorff's method in the manner described above. (See Fig. 1.)

1. *Balance between the E. M. F. of the Standard cells and the difference of potential over the manganin resistance, due to the current necessary to counteract the twisting moment of the wire.*

As the source of the current through the instrument we used a storage battery with a voltage of about 72 volts. In the following tables we have given the experiments in the order in which they were taken. In the first column is given the arrangement of the coils. In arrangement 2 the current was flowing through the movable coil in the direction opposite to that which it had in the arrangement 1, while the direction of the current through the stationary coil remained the same in both arrangements. This was done to eliminate the error due to the lead wires between the two coils. The difference in our results for the balancing current in the two cases is small, but quite apparent. In the second column the number of ohms necessary for a balance with the current through the whole instrument in one direction is given, in the third the same with the current reversed; these two currents correspond to i_1, and $-i_2$ or i_3 and $-i_4$ of equation (11).

The fourth column shows the number of ohms necessary to balance the E. M. F. of the two Carhart-Clark cells, the following, the E. M. F. of the cells, supposing the value at 24° to be 1.435 volts. The following two columns contain the temperatures of the electrodynamometer (t_d) and the cells (t_c) respectively. The value

TABLE VIII.

Arrangement.	R_a	R_b	R_1	E. M. F.	t_d	t_c	I''
1	7901.5	7773	4944.1	2.86878	26°.35	24°.95	.98284
2	7897	7765.7	4944.1	2.86878	27 .8	24 .95	.98246
2	7896	7764.6	4942.9	2.86795	28 .09	25 .6	.98236
1	7902	7773	4942.8	2.86744	26 .68	26 .0	.98274
2	7898.6	7770.7	4943	2.86744	26 .95	26 .0	.98242
2	7902.6	7773.4	4944.9	2.86808	26 .03	25 .5	.98242
1	7904	7774.1	4944.85	2.86795	27 .39	25 .6	.98284

of the manganin resistance was taken as 4.626 ohms. The current I'' corresponding to zero earth's field and reduced to 27° of the electrodynamometer is given in the last column and shows, for the same arrangement, no larger discrepancy than one in ten thousand.

The average value 0.98261 amp. for the two arrangements is greater than the calculated value 0.98158 in the ratio 1.00105. This ratio can be used later to correct the value found for the electrochemical equivalent.

Since we were not sure of the temperature coefficient of the Carhart-Clark cells at the high temperatures at which they were used, a separate experiment was made to find the variation of their E. M. F. with temperature. A Clark cell (United States Standard) was always kept at a constant temperature and the temperature of the experimental cells changed. No observations were taken immediately after a change, in fact in general, the temperature had been practically constant for several hours. Supposing the E. M. F. of a Carhart-Clark cell to be 1.435 V. at 24° C. we found the following values.

TABLE IX.

Temp.	E. M. F.
22°	1.43628 volts.
23	1.43564
24	1.43500
25	1.43436
26	1.43372

These values have been used in Table VIII.

Instead of calculating I'' we may correct the resistances R_a and R_b to the values they would have had, if the electrodynamometer had been at 27°, using the temperature coefficient, found for the wire, and correct R_1 to 24° of the cells.

TABLE X.

Arrangement.	R_a	R_b	R_1
1	7900.4	7771.9	4946.3
2	7898.4	7767.1	4946.3
2	7897.8	7766.4	4946.4
1	7901.5	7772.5	4947.2
2	7898.5	7770.6	4947.4
2	7901.0	7771.8	4948.2
1	7904.5	7774.6	4948.3

Calling R_a' and R_b' the resistance found with the arrangement 1, R_a'' and R_b'' those with arrangement 2, we find as the resistance

necessary for a balance, if there were no influence of the earth's magnetism or asymmetry in the instrument,

$$R_2 = \sqrt{R_a' R_b' R_a'' R_b''}$$

The values of R_1 and R_2 calculated from the numerical results are

$$R_1 = 4947.2 \text{ and } R_2 = 7835.5$$

therefore

$$I = 1.58383 \frac{E}{R} \text{ amp.} \qquad (1a)$$

2. *Balance between the E. M. F. of the Standard cells and the difference of potential due to the silver depositing current.*

The method of procedure during these experiments was the following: As in the preceding we used the storage battery, having about 70 ohms in series with the two voltameters. Before sending the current through the voltameters it was allowed to flow through the resistance, in order to heat it up, and a preliminary balance was taken. Then by means of a switch the silver voltameters were thrown in series with the rest of the circuit. By means of this precaution we obtained a very constant current and the small changes taking place were so regular that there was no difficulty in taking an average of the resistances necessary for a balance during the hour the current was running. In taking the average we were careful to allow for the time-interval between consecutive balances. Out of a number of observations we have selected four in which the changes in the resistances were small, on the average about 20 ohms in 7500 ohms during the hour, and in which the temperature of the Standard cells was nearly 24°. The change of the E. M. F. of the Standard cells never exceeded 2 in 50000 during the whole hour.

The rate of the clock was repeatedly compared with the rate of the Riefler clock and it was found that one hour of the Riefler clock was equal to 3601.3 seconds of the other.

In the first two of these determinations two Standard Carhart-Clark cells were used, whose E. M. F. was a little smaller than that of the cells used in the rest of the experiments. The ratio was found to be 1.00243. In the following table R_2' is the average resistance necessary to balance the current and R_1' the resistance to balance the E. M. F. of the cells. In the fourth column are given the cor-

rected values of R_1', *i. e.*, the resistances that would balance the cells if they were at 24° C.; in the following the temperature of the cells and in the last the electro-chemical equivalent worked out under the supposition that the values of E (2.870 volts) and R (4.626 ohms) are correct.

In the first case the duration of the experiment was only 40 minutes, in the rest, 60 minutes.

TABLE XI.

Date.	R_2'	R_1'	R_1' corr.	t	Silver deposit.	z
June 4th.	7622.15	4830.71	4842.6	24°.14	2.6211 grms.	1.11795 10^{-3}
" 19th.	7529.25	4911.05	4919.2	22 .3	3.82301	1.11789
" 27th.	7530.69	4938.79	4938.4	23 .82	3.8094	1.11807
" 30th.	7664.86	4943.75	4944.9	24 .5	3.8718	1.11796
					Mean value =	1.11797×10^{-3}

Multiplying the mean value for z, 0.00111797 grm., by 1.00105 (the ratio found above) we obtain $z = 0.0011192$ grm. per ampère per second.

Taking the mean of R_1', R_2' and q, the silver deposited in one hour, we obtain

$$R_1' = 4911.3 \text{ ohms.}$$
$$R_2' = 7586.7 \text{ ohms.}$$
$$q = 3.8589 \text{ grms.}$$

Substituting these values and those obtained above for I_1, R_1 and R_2 in our formula (5)

$$z = \frac{q}{It}\frac{R_1' R_2}{R_1 R_2'}$$

we find as the electrochemical equivalent of silver

$$z = \frac{3.8589 \times 4911.3 \times 7835.5}{.98158 \times 3601.3 \times 4947.2 \times 7586.7} \text{ grms.}$$

$$z = 0.0011192 + \text{ grm. per ampère per second.}$$

This is about 1/9% larger than the accepted value. It agrees with the average value of Pellat and Potier and with the value Kahle found when using an old solution of silver nitrate. Kahle gives for

this, in his preliminary report (Zeitschr. f. Instr., XVII., 144, 1897), a value almost identical with ours, 1.1193 mgrm. Our experiments with the silver voltameters were taken with a solution that had been used three or four times before. Moreover it is of interest to note that the discrepancy between Rowland's and Griffith's value of the mechanical equivalent of heat disappears, when we use as the unit of current one about .11% larger than the ampère.

Glazebrook found as the E. M. F. of the Clark cell 1.4342 volts at 15°, using as the electrochemical equivalent of silver the accepted value 0.001118 grm. A recalculation of the E. M. F. on the basis of the value found by us gives

$$E = 1.4327 \text{ volts at } 15°\text{C.}$$

Kahle's value is 1.4328 volts.

Kahle has found that the E. M. F. of a Standard cell is more constant and gives better results than a silver voltometer, though the latter will always be the ultimate standard.

With our apparatus we can easily determine the E. M. F. of a Standard cell by direct comparison, knowing accurately the resistance of the manganin coil. It is intended to continue the experiments with special reference to the E. M. F. of the Clark cell.

In conclusion we wish to express our sincere appreciation for the substantial recognition accorded to our work by the American Association for the Advancement of Science and to thank Professor Carhart for the cells he allowed us to use and the great interest he has shown in the progress of our investigation at all stages.

Since the above was printed Kahle has published his investigations on the use of the silver-voltameter,[1] and we are now able to compare more accurately his results with ours. There are three important points to be considered.

1. The solution used. What Kahle terms an "old" solution is one originally neutral, which became more and more acid, as it was employed in the voltameters. Larger deposits were obtained with this than with the "fresh" solution, the difference often exceeding one part in one thousand. On addition of Ag_2O this discrepancy

[1] Zeitschr. f. Inktrum-Kunde, Aug. and Sept., 1898.

diminished considerably. Since we used from the beginning the silver-oxide to insure neutrality, we did not observe the irregularities in crystallization and color of the deposited silver, that are so characteristic in his experiments. Nevertheless the addition of the oxide seems to produce a deposit about five parts in ten thousand larger than what Kahle found with his standard solution.

2. Method of washing the solution. Kahle observes in general that with warm water and correcting for the solution of silver in the warm water the deposit is about one to two parts in ten thousand smaller than what he obtained by a treatment with cold water Our method of letting the crystals soak in cold water for a whole day seems to be more thorough than Kahle's, who rinsed the deposit three or four times until he was unable to show the presence of silver in the water by means of HCl.

3. Platinum or silver cathode. Kahle found that the silver deposit was larger, if previously some silver had been deposited on the cathode. It stands to reason that this difference is relatively the more apparent the shorter the time of electrolytic action and the smaller the mass of silver deposited during the experiment, and we can not expect as large a relative discrepancy, since we deposited about four times as much silver as Kahle. In fact, in this particular point we cannot corroborate Kahle's results. In the first two observations of Table XI. the silver was deposited on silver, in the last two on the clear platinum cathodes. We believe that the influence of moisture during the weighing may be sufficient to explain the discrepancies in our results, though we were careful to weigh in closed balance cases, the air of which was dried by calcium-chloride.

For a final comparison, taking into account the first two points mentioned, we have therefore to add to Kahle's value for the electrochemical equivalent of silver (0.0011183 grm.), about six or seven parts in ten thousand. This would make Kahle's value 0.001119 grm. as compared with our 0.0011192 grm.

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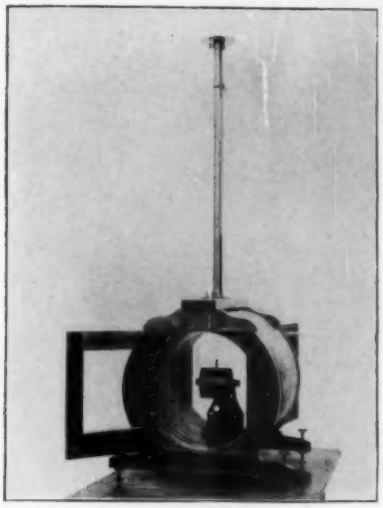

FIG. 3.
General View.

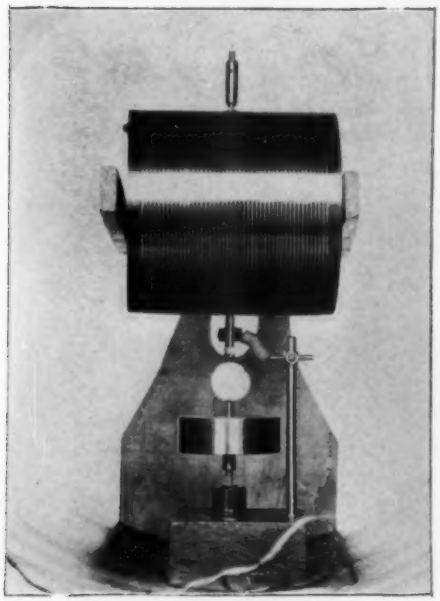

FIG. 4.
Suspended Coil.
Patterson and Guthe: Electrochemical Equivalent of Silver.
[Views of the Electrodynamometer.]

THE VELOCITY OF LIGHT IN THE MAGNETIC FIELD.

By Henry T. Eddy, Edward W. Morley, and Dayton C. Miller.

Part I.

Upon the Electromagnetic Theory of the Rotation of the Plane of Polarization in a Magnetic Field.

By Henry T. Eddy.

IN a note upon this subject read before Section B at the Toronto meeting of the A. A. A. S., August, 1889,[1] the present writer developed the essential points in the present paper with the exception of the numerical estimate of the amount of the suspected effect.

At the same meeting[2] Professor Morley pointed out a method (See Part II) by which it might be possible to demonstrate experimentally the existence of the suspected effect, which consisted in a decrease in the velocity of the light passing through any magnetic field in a medium causing rotation of the plane of polarization.

An appropriation was generously granted from the funds of the A. A. A. S. to construct apparatus in accordance with the proposal of Professor Morley for the purpose of trying to discover, if possible, whether such an effect as theory indicates was, in fact, observable.

The report finally presented to the A. A. A. S. at their Boston meeting, August, 1898, shows that no such effect was observable with the apparatus employed. The numerical estimate at the end of this paper makes it evident that the effect to be looked for was only two one-hundred-millionths of a wave-length, while the smallest effect capable of observation was five one-hundredths of a wave-length. Hence the effect to be looked for was only four one-millionths of the smallest effect capable of observation with the apparatus. This sufficiently accounts for the failure to observe any

[1] Proc. A. A. A. S., 1889, Vol. 38, p. 129. [2] Ibid., p. 130.

such effect. But it was not known at the time of observation what might be the possible magnitude of the effect.

The effect is, therefore, one which would, in the field employed in this apparatus, amount to about one one-thousandth of an Ångström unit with sodium light.

It is the hope of the writer that the effect in question may not prove to be finally beyond the reach of experimental demonstration by spectroscopic methods, which, as stated in the note above referred to, seem, perhaps, the most promising for rendering the effect observable.

The differential equations which were proposed by Professor Rowland[1] to express the propagation of plane polarized light as an electro-magnetic disturbance in a magnetic non-conducting field in the direction of the lines of force taken as parallel to the axis of z, are :

$$\left.\begin{aligned} K\mu\left(\frac{d^2F}{dt^2} - \frac{c\gamma}{4\pi\mu}\cdot\frac{d^3G}{dtdz^2}\right) &= \frac{d^2F}{dz^2} \\ K\mu\left(\frac{d^2G}{dt^2} + \frac{c\gamma}{4\pi\mu}\cdot\frac{d^3F}{dtdz^2}\right) &= \frac{d^2G}{dz^2} \end{aligned}\right\} \qquad (1)$$

in which F and G express the electro-magnetic momenta in the directions of x and y respectively, and they are so named because they are defined, in case $c = 0$, with respect to the actual electromotive forces P and Q per unit of length in the directions of x and y by the equations :

$$dF/dt = -P \quad \text{and} \quad dG/dt = -Q,$$

and these equations are analogous to the mechanical equations for forces and momenta.

K is the specific inductive capacity of the medium, μ is its magnetic permeability, $\mu\gamma$ is the magnetic induction parallel to the axis of z, and c is the coefficient of proportionality for the Hall effect. There is no electromotive force along z, and there is no magnetic induction except along z.

In case $c = 0$, eqs. (1) reduce to those originally proposed[2] by Maxwell.

[1] Am. Jour. Math., 1880, vol. 3, p. 109.

[2] Maxwell's Elect. and Mag., Vol. 2, 3d ed., p. 439.

Rowland says that we may well suppose one solution of eqs. (1) to be :

$$\left.\begin{aligned} F &= r \cos (nt - qz) \cos mt \\ G &= r \cos (nt - qz) \sin mt \end{aligned}\right\} \quad (2)$$

in which $\cos (nt - qz)$ is the factor on which depends the transverse vibration of the electromagnetic disturbance which is propagated with a velocity v, a wave-length λ, and a period of oscillation T, such that

$$v = n/q, \quad \lambda = 2\pi/q, \quad T = 2\pi/n. \quad (3)$$

The periodic factors $\cos mt$, and $\sin mt$ are those which would express a continuously progressing rotation of the plane of polarization directly proportional to the time but independent of space. So that as time passes the rotation continues without cessation though space be arbitrarily assumed to be constant.

While it might be possible to arrange an apparatus for experiment in which such a continuously progressive rotation of the plane of polarization could be imparted mechanically to that plane at the place where the light enters the medium, nevertheless it is evident that that is not what is intended to be treated by eqs. (2).

The writer would therefore propose instead, the following solutions as applicable to the case in hand, viz., the case of a stationary plane of polarization which is twisted in its passage through the medium in the magnetic field. It is this stationary twist which is described by the word "rotation" used in this paper. The eqs. proposed are :

$$\left.\begin{aligned} F &= r \cos (nt - qz) \cos pz \\ G &= r \cos (nt - qz) \sin pz \end{aligned}\right\} \quad (4)$$

in which the factors $\cos pz$ and $\sin pz$ express a continuously progressive twisting of the plane of the polarization increasing uniformly with the space traversed by the ray. If the angle of twist be θ, and the distance in which the plane makes one complete turn be σ, then

$$\theta = pz, \quad \sigma = 2\pi/p = 2\pi z/\theta, \quad \text{and } \sigma/\lambda = q/p. \quad (5)$$

In order to obtain the equations of condition which must exist between the constants n, q and p in order that eqs. (4) may be solutions of eqs. (1), let the values of F and G be substituted in eqs. (1). The result may then be written in the form :

$$\left.\begin{aligned} A\cos(nt-qz)\cos pz + B\sin(nt-qz)\sin pz = 0 \\ A\cos(nt-qz)\sin pz - B\sin(nt-qz)\cos pz = 0 \end{aligned}\right\} \quad (6)$$

in which the values of A and B are those given in eqs. (7).

Eqs. (6) are identically equal to zero for all values of t and z, and eqs. (4) are consequently solutions of eqs. (1), when and only when $A = 0$ and $B = 0$. Hence, writing the values of A and B in full and equating these values to zero we have:

$$\left.\begin{aligned} q^2 + p^2 - K\mu n^2 - Knqp\frac{c\gamma}{2\pi} = 0 \\ 2qp - Kn(q^2+p^2)\frac{c\gamma}{4\pi} = 0 \end{aligned}\right\}. \quad (7)$$

By taking the sum and difference of eqs. (7):

$$\left.\begin{aligned} K\mu n^2 = (q-p)^2(1+ne) \\ K\mu n^2 = (q+p)^2(1-ne) \end{aligned}\right\} \quad (8)$$

in which eqs. for brevity,

$$e = K\frac{c\gamma}{4\pi}. \quad (9)$$

In eq. (9), since c is small, e is small also.

Now, in eq. (5) σ is very large, compared with λ, hence q is very large, compared with p. The positive sign must, therefore, be taken in extracting the square roots of eqs. (8); hence by the binomial theorem

$$\left.\begin{aligned} n\sqrt{K\mu} = (q-p)(1+ne-\tfrac{1}{2}n^2e^2+\text{etc.}) \\ n\sqrt{K\mu} = (q+p)(1-ne-\tfrac{1}{2}n^2e^2-\text{etc.}) \end{aligned}\right\}. \quad (10)$$

By taking the sum and difference of eqs. (10) we have approximately

$$\left.\begin{aligned} n\sqrt{K\mu} = q(1-\tfrac{1}{2}n^2e^2) - pne \\ 0 = qne - p(1-\tfrac{1}{2}n^2e^2) \end{aligned}\right\}. \quad (11)$$

From the last of eqs. (11) $p = qne$ nearly, and this substituted in the first of eqs. (11) gives:

$$n\sqrt{K\mu} = q\left(1-\tfrac{3}{2}\frac{p^2}{q^2}\right). \quad (12)$$

If $v_0 = 1/\sqrt{K\mu}$ be the velocity of light in the medium when the magnetic field vanishes, we have by eqs. (3) and (5):

$$v = v_0 \left(1 - \tfrac{3}{2} \frac{\lambda^2}{\sigma^2} \right) \tag{13}$$

$$\therefore \frac{\delta v_0}{v_0} = \frac{v_0 - v}{v_0} \quad \tfrac{3}{2} \frac{\lambda^2}{\sigma^2}. \tag{14}$$

If z_0 be the length of path in the medium occupied by a certain number of wave-lengths when the field vanishes, and $z_0 - \delta z_0$ be the length of path occupied by the same number of wave-lengths when the field has diminished the velocity,

then, $$\frac{\delta v_0}{v_0} = \frac{\delta z_0}{z_0} = \tfrac{3}{2} \frac{\lambda^2}{\sigma^2} \tag{15}$$

and $$\frac{\delta z_0}{\lambda} = \tfrac{3}{2} \frac{\lambda z_0}{\sigma^2}. \tag{16}$$

Eq. (16) expresses the fraction of a wave-length by which light is retarded in traversing a length of path z_0 through the field in the medium.

Eqs. (13) to (16) state that the assumption of eq. (4) as the solution of eqs. (1) leads to the result that the velocity of light is less while the plane of polarization is being twisted in a medium by a magnetic field than when the magnetic field vanishes and the plane undergoes no twisting.

In case we take the wave-length of sodium light as approximately $\lambda = 6 \times 10^{-5}$ cm., and let $z_0 = 120$ cm., and $\sigma = 240$ cm., as they were in the apparatus employed, we find from eq. (15).

$$\delta z_0 = 11 \times 10^{-12} \text{ cm.}$$

or approximately one one-thousandth of an Ångström unit, which last is 10^{-8} cm.

$$\therefore \quad \delta z_0 / \lambda = 2 \times 10^{-7} \text{ nearly,}$$

or two one-hundred-millionths of a wave-length of sodium light is the total amount of the retardation of sodium light during its passage through the apparatus with which the experiment was made:

and $$\therefore \quad \delta v_0 / v_0 = \delta z_0/ z_0 = 10^{-13} \quad \text{nearly,}$$

or the retardation to be looked for in the field employed is one part in one hundred thousand times one hundred million.

Professor Rowland has shown in his discussion of eqs. (2) that they involve an increase in the velocity of light instead of a decrease such as follows from eqs. (4); and in the further discussion of the matter he has assumed that he is at liberty to write the equation $\theta = mz/v$, whereas in his equations $\theta = mt$ is the only value of θ which is admissible, since z and t are independent. It would seem, therefore, that so much of his discussion as depends upon this assumed value of θ may not be valid.

MINNEAPOLIS, OCTOBER, 1898.

PART II.

EXPERIMENTS ON THE VELOCITY OF LIGHT IN A MAGNETIC FIELD.[1]

BY EDWARD W. MORLEY AND DAYTON C. MILLER.

At the Toronto meeting of the American Association for the Advancement of Science in 1889, Professor Henry T. Eddy (see Part I), discussed the partial differential equations of the motion of plane polarized light in a magnetic field. These equations contain terms "expressing the transverse electromotive force due to the Hall effect. The particular solution of these equations which Rowland proposes contains a periodic factor dependent upon the time." In Professor Eddy's paper, "a different particular solution, containing a periodic factor dependent upon the space which the ray traverses in the field, is discussed at length and compared with the solution proposed by Rowland." It is shown that the velocity of the ray would be increased if there is a periodic factor dependent upon the time, but would be decreased by a factor dependent upon the space.

In the discussion which followed the reading of Professor Eddy's paper, one of the present writers suggested a form of apparatus which would detect the suspected change of velocity, provided it amounted to one part in fifty millions. The whole matter was of

[1] Report of an experiment made with the aid of a grant from the research fund of the American Association for the Advancement of Science.

such interest that the section of the Association before which the paper was read, obtained, from the research funds of the Association, a grant of money with which to construct the apparatus and make the experiment. The apparatus was finished in the summer of 1890, and Professor Eddy came to Cleveland to take part in the experiment.

The optical part of the apparatus consists of the interferential refractometer used by Michelson in his experiments to determine

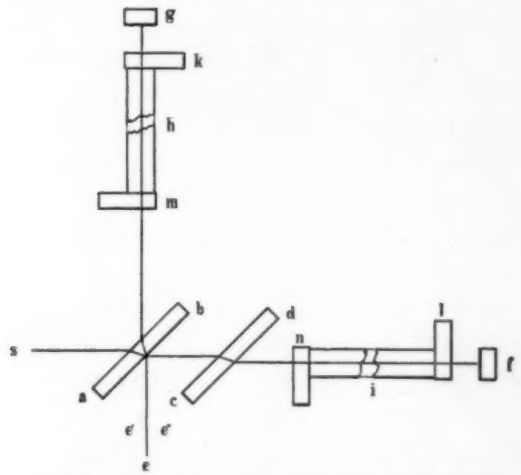

Fig. 1. Diagram, showing arrangement of optical parts.

whether light moves with the same velocity in different directions in the solar system. A train of waves of light from the source *s*, (Fig. 1) is divided at the second surface of *ab*, which is coated with silver of such thickness that the division is nearly equal. If the mirrors, *f* and *g*, are properly adjusted, the two parts reunite at *ab*, and interference bands can be seen by the eye at *e*, if the distances of *f* and *g* from *ab* are properly related. If, now, any force acts intermittently to change the velocity of one part of the ray while the paths are separated it may be detected by a motion of the interference bands, provided the change is not too small.

If we put two tubes, *h* and *i*, in the paths of the divided ray, make these of very nearly the same length, close them with plane parallel glasses of equal thickness, and fill the tubes with carbon bi-

sulphide, we shall still obtain interference bands. If we surround each tube with a coil of copper wire, use polarized light, and close the circuit through one coil, there will be a rotation of the plane of polarization of the ray which passes through the magnetic field. If the rotation be not so much as to prevent, or too much affect, the interference of the parts of the divided train of waves, a change of velocity of light while in the magnetic field will produce a displacement of the interference bands. If we complete the circuit in the two coils alternately, we double the displacement which is sought to detect.

If any displacement is observed, it will be necessary to determine whether it may not be due to strain or displacement of parts of the apparatus. In the case of the plates *ab*, *cd*, *f* and *g*, this may be accomplished by watching the effect of closing alternately the circuits through the two coils while tubes *i* and *h* contain only air. But a disturbance which should affect the position or length of one of these tubes needs special means for its detection. The plates which close the ends of these tubes are accordingly placed as shown at *k*, *l*, *m* and *n*, and are silvered except where they cover the tubes. When proper adjustments are made, the observer can see interference bands through the tube by placing the eye at *e*, interference bands at the covers of the nearer end of the tubes by placing the eye at e', and interferences at the covers of the farther ends of the tubes by placing the eye at e''. Since the interferences produced in these three cases are due to rays which have passed through only glass and air, they can be so adjusted as to detect with ease and certainty a displacement which would be entirely insensible in the irregular and most unstable interference bands produced after a ray of light has passed through a liquid so optically unstable as carbon bisulphide. Our means of detecting the effect of mechanical disturbances were therefore greatly superior to those of detecting the change of velocity in question, and the statement that no displacement of interference bands was caused by mechanical disturbance due to the opening and closing of the current through the coils is open to no doubt.

The plates *ab* and *cd* were 9.3 by 2.0 cm.; *f* and *g* were 2.0 cm. by 2.0 cm.; *k*, *l*, *m* and *n* were 4.3 cm. by 2.0 cm. The tubes *h*

and *i* were 1.7 cm. in inside diameter, and were 38.1 cm. long. They lay in adjustable supports, and were not in contact with the coils which surrounded them. The core around which the wire of the coils was wound was rectangular in section. Around a free space of 4.2 cm. by 2.2 cm. was first a nonconducting layer of 0.6 cm. in thickness, then a space 0.6 cm. thick for the circulation of water; and, lastly, a coil of wire. Each coil was made of two wires laid side by side, so that when the same current was sent through the two wires in opposite directions, its effect was null. A commutator was so arranged as to send a constant current through one wire of each coil, while the same current was so sent through the other wires of the two coils as to neutralize the effect of the current in the first wire of one coil, and to double it in the other coil. The heating due to the passage of the current was therefore identical in the two coils. The wire of each coil weighed about 82 kilogrammes, and the length covered by the wire was 30 cm.

The flame of a Bunsen lamp was colored by the introduction of some sodium compound as the usual source of light, because the interference bands can, in homogeneous light, more easily be given forms which permit delicate discrimination; a luminous gas flame and the electric arc were also employed. A Nicol prism was sometimes interposed, and the plane of polarization was placed in various positions. The optical parts of the apparatus were so related that the visible interference bands coincided with the surface of the more distant mirrors; their displacement can then be detected by referring their position to lines ruled on the mirrors.

With this apparatus, one observer adjusted the interference bands in width and position; when these were made suitable, the second observer used the commutator to produce a magnetic field, first in one coil and then in the other. He announced the change from one to the other in such a way that the first observer did not know which coil was acting, so that the effect of any possible bias might be excluded.

With this apparatus, both observers suspected a slight displacement of the interference bands, and both satisfied themselves that this displacement was in the direction which indicated acceleration in the light passing through the magnetic field. The amount of

the acceleration thus suspected was not more than one part in one hundred millions. But the displacement was so slight that they could not be confident of its existence.

In the examination of the effect of a magnetic field on a ray passing through it, the appearance on making and breaking the circuit suggested the possibility of a change in the density of carbon bisulphide while in the field. There was produced a momentary displacement of the interference bands, which was unmistakable, being many times as great as the displacement which it was suspected continued as long as the current passed. It was easy to imagine that the force which quickly brought the molecules of the liquid into the new positions in which they produce rotation of the plane of polarization, might carry them beyond the position of equilibrium, to which they would quickly return. To examine this phenomenon, a thick brass box filled with carbon bisulphide was placedin each coil, and the two boxes were connected with each other and with a glass tube of small diameter. After all parts of the apparatus had come to a uniform temperature, the level of the liquid was brought to a convenient part of the glass tube, and a current of twenty-seven ampères was sent through the two coils, in such direction as to produce a magnetic field in both coils. Each time the circuit was completed, there was a momentary rise of the liquid by 0.06 mm., and an instantaneous fall to 0.03 mm. On breaking the circuit, there was a momentary fall to —0.03 mm., and an instantaneous rise to 0.00 mm. This displacement could be well measured three or four times by means of a reading micrometer with an amplification of about sixty diameters, the probable error of a reading being less than 0.005 mm. After three or four measurements, the heat developed in the coils made it necessary to wait many hours for equilibrium of temperature to be established again. There were 380 cubic centimeters of carbon bisulphide in the magnetic field; the area of the capillary tube was 0.55 square millimeters; the lasting change of density or the change in the volume of the containing envelope was therefore one part in twenty-three million. The intensity of the field was 1,650 centimeter-gramme-second units.

This matter was afterward investigated with another brass box

made to fit loosely between the poles of a dynamo-electric machine, whose coils had been so connected that the box was placed between a north and south magnetic pole. The material of this box was free from magnetic admixture, and was supported so as not to be in contact with any part of the dynamo. In this envelope, we have not yet been able to detect any such change of density or volume as was suspected before. The cause of the momentary displacement of the interference bands which was so obvious will therefore need further examination.

At the meeting of the American Association for the Advancement of Science in Indianapolis, in 1890, Professors Morley and Eddy made a report of the result of their experiment. The matter seemed of such interest to the section that it obtained a further grant with which to continue the experiment with more powerful apparatus. Owing to other occupation, the construction of this apparatus has been delayed till the present year. With the assistance of Professor Dayton C. Miller, now associated with Professors Eddy and Morley, a new apparatus has been set up with which the acceleration or retardation produced would be three times as great as with the former apparatus. Further provision has been made for securing much greater thermal and optical stability of the carbon bisulphide used in the apparatus. As a result of these two modifications, our power of detecting small changes of velocity of light in a magnetic field is probably five times as great as in the case of the former apparatus.

The second apparatus differs from the former in two respects. The optical parts remain as before; the only difference in their use depends on the fact that the coils are now twice as long as before, so that the mirrors *f* and *g* are farther from the mirror *ab*. The tubes *i* and *h* are therefore 30 cm. longer and the column of carbon bisulphide in the magnetic field is twice as long as at first. Further, these tubes are so connected that a current of bisulphide can be passed through them at pleasure; and around them are concentric tubes through which passes a current of water, intended to prevent the heating of the bisulphide by the passage of the electric current through the coils. A tank, placed in the room with the apparatus, supplies water at constant temperature, for this purpose; in this

tank is immersed another tank, which supplies a current of bisulphide having the same temperature as the water. This arrangement for preventing changes of temperature in the column of carbon bisulphide in the tubes of the apparatus greatly increases the steadiness of the interference bands to be scrutinized, and so increases the delicacy of the observations.

The illustration taken from a photograph shows the apparatus ready for use. At the extreme right are seen the commutator, ampèremeter, and resistance coils, used in managing the electric current. The wooden stand at the extreme left carries the source of light and the condensing lens. The adjacent stone pier carries the coils; between them is seen the cubical block of stone which supports the diagonally placed mirrors *ab* and *cd* of Fig. 1. Apparently just above this block, but really some yards to the rear, is a double tank supplying water and carbon bisulphide to the apparatus. The reading telescope with which the observations were made is marked by the hanging cloth. On the left edge of the pier is seen an iron stand carrying a Nicol's prism for polarizing the ray of light sent through the apparatus; on the wooden stand to the right of the pier is seen the analyzer, by means of which the rotation produced was measured while the current was adjusted so as to secure the rotation desired.

When a current of twenty-seven ampères is sent through the coils of this apparatus, there is produced a rotation of the plane of polarization of light by one-half a circumference; in the previous apparatus, we could utilize a rotation no larger than about fifty degrees of arc. The rotation utilized now is therefore three and one-half times as great as that in the former case, involving a corresponding increase in the change of velocity which it is hoped to detect.

With this apparatus, a current was sent through the coils at the call of the observer at the telescope. The current had been adjusted so that it should produce a rotation by half a circumference of the plane of polarization of the light employed at the time. This rotation caused the interference bands, which had been seen before the passage of the current, to shift their position by half a wave-length; this was due to the reversal of phase of the ray which had

suffered rotation. A micrometer wire was set on a maximum or a minimum, the current was reversed, and the coincidence of the wire with the maximum or the minimum was scrutinized. As a result of such scrutiny, often repeated, both observers are confident that there was no displacement amounting to one-twentieth of a wave-length. They also are of opinion that there is no advantage to be expected from an increase in the size of the apparatus, and a repetition of the experiment with carbon bisulphide, unless it shall be kept thermally and optically homogeneous by rapid motion. If dense glass, sufficiently transparent in a long cylinder, could be used in place of the bisulphide, it would probably be possible to detect a quantity which, if a liquid is employed, could not be detected except at an expense which would be prohibitory.

The result reported at the Boston meeting of the American Association is, that we are confident that when light corresponding to the solar D line is passed through one hundred and twenty centimeters of carbon bisulphide in a magnetic field which produces rotation by half a circumference in the plane of its polarization, there is no such change of velocity as one part in sixty million, and probably no such change as one part in a hundred million.

A PHOTOMETRIC STUDY OF THE SPECTRA OF MIXTURES OF GASES AT LOW PRESSURES.

BY ERVIN S. FERRY.

THE enormous difficulty of obtaining a specimen of a gas so pure and a vacuum discharge tube so clean that no extraneous spectrum will appear when the gas is rendered luminous by a current of electricity, is a fact familiar to all workers in spectroscopy. The prime importance of this subject is rendered evident when we attempt quantitative analysis of substances by means of their emission spectra, and also when we attempt to construct hypotheses to explain questions of such fundamental importance in physics as the differences in the spectra at the two electrodes of a vacuum tube, and the appearance of what are called "multiple spectra." The whole question of the division of the energy among the constituents of a luminous gas mixture is a subject that has received slight attention, and has been the subject of little experimentation.

The object of this research has been to study the spectra of mixtures of two gases that have no chemical action upon each other, under different conditions of proportion, pressure, and strength of exciting current. The subject of this research and the method employed were suggested by Professor Knut Ångström, to whom it is my agreeable duty to acknowledge the deepest obligations for advice and assistance throughout the entire course of these experiments.

The apparatus employed consisted of a two-prism spectroscope by Steinheil, of Munich, furnished with a Vierordt double slit by Krüss, of Berlin, a discharge tube kept in connection with a mercury pump, a battery of 1,200 accumulator cells by Klingelfuss, of Basel, for the purpose of rendering luminous the gas in the discharge tube, a d' Arsonval galvanometer by Carpentier, of Paris, for measuring the amount of current flowing through the discharge tube, and an Argand burner in connection with a gas pressure regulator to furnish the standard spectrum of contant brightness. The method of producing clear discharge tubes and pure gases, as well as the general

arrangement of apparatus, were the same as that described in the first part of this paper.[1]

Although it is customary to assume that the brightness of any part of the continuous spectrum from a standard comparison light is directly proportional to the area of the collimator slit through which the light passes, it is quite probable that this relation is not strictly true under all conditions. From theoretical considerations we would expect that when the light sources are so different in intensity as to require the two collimator slits to be of greatly different area in order to bring about an equality of luminosity of the two spectra in the eye-piece of the telescope, that there might be deviations from this law. If the luminosity curves of the two light sources are not parallel at the part of the spectrum being studied, the areas of the slits will not be directly proportional to the luminosities of the two lights.[2] Again, if one slit is much wider than the other, the difference in purity of the two spectra will be so different that an accurate comparison of the luminosities will be very difficult. The applicability of the law to the conditions of the present experiment was determined in the following manner. One slit was set at constant width in front of a light of constant intensity. Interposed between the other slit and other light was a diaphragm containing four holes of nearly equal area. The light going through each hole was measured, as well as that through each two holes and that through all four holes. The readings thus obtained show that under the conditions of this experiment, the departures from the law are less than the errors of observation.

In the first part of this paper it was shown that in the case of a pure gas at low pressure, the luminosity of each spectal line varies directly as the exciting current—at least within the limits of the experiment. It was also shown that in the case of pure gases rendered luminous by a current of constant strength, the luminosity of a spectral line increases in a regular manner as the pressure decreases. A considerable part of the present series of experiments has been to determine if these laws also apply to a mixed gas. The method employed consisted in measuring the luminosity of the spectral lines in a mixture of two gases in known proportion for various pressures,

[1] PHYS. REV., VII., p. 1, 1898.

[2] D. W. Murphy, Astrophysical Journal, VI., p. 1, 1897.

and for various strengths of exciting current. The two gases selected for study were hydrogen and nitrogen. This selection was made on account of their lack of chemical action upon each other and on account of their comparative ease of preparation in a pure state. A definite proportion of the two gases was obtained by the following device: a glass bulb closed at the top but open at the bottom was filled with water and weighed while immersed in water: a few bubbles of one gas were introduced and the displaced water determined; some bubbles of the second gas were then introduced and the water thereby displaced determined. From these data was computed the percentage composition of the mixture. The mixed gas was introduced into the discharge tube through a barometric column in the manner devised by Cornu.

It is a matter of common observation that the pressure of a vacuum tube containing certain gases decreases when a current is passed through the gas for a considerable length of time. Nitrogen shows this effect in the greatest degree. One of the first observers of the phenomenon was A. Morren [1] who surmised that the diminution of pressure was due to the presence of mercury vapor in the tube. From a series of very careful experiments on nitrogen, Threlfall [2] has demonstrated that mercury vapor is the principal cause for the diminution of pressure in the case of that gas. He concludes that it is brought about by the formation of a compound of nitrogen and mercury. Throughout the course of the present series of experiments the mercury lines were invisible. After a specimen of the gas had been admitted to the apparatus, a period of not less than one hour was allowed for diffusion to take place before observations were taken. The observations were then made as quickly as possible. Before taking readings at a different pressure, the tube was evacuated and a fresh portion of the mixture introduced. By this means, it is certain that the composition of the mixture could not change during a set of observations by an amount sufficient to influence the readings. The pressure measured by a MacLeod manometer at the beginning and at the end of a set of observations never differed by more than three or four hundredths of a millimeter of mercury.

[1] "Sur la conductibilité électrique des gaz sous de faibles pressions." Ann. de Chem. et de Phys., IV., 4, p. 331 (1865).

[2] Phil. Mag., V., 35, p. 1 (1893).

Table I.

Hydrogen line $\lambda = 6563$.

Pure Hydrogen.					
Pressure in mm.	3.8	2.7	1.4	0.7	
Luminosity. Current = 2 M. A.	4	6	8	10	
" = 4 "	8	12	16	20	
" = 6 "	13	18	23	29	
95 % Hydrogen.					
Pressure in mm.	3.32	2.24	1.75	0.83	0.35
Luminosity. Current = 2 M. A.	5	6	7	10	15
" = 4 "	9	12	14	20	29
" = 6 "	15	19	22		
64 % Hydrogen.					
Pressure in mm.	1.73	0.8	0.51	0.35	
Luminosity. Current = 2 M. A.	4	6	7	9	
" = 4 "	8	13	15	17	
57 % Hydrogen.					
Pressure in mm.	0.75	0.39			
Luminosity. Current = 2 M. A.	4	7			
" = 4 "	8	15			

Nitrogen band $\lambda = 6465$.

Pure Nitrogen.					
Pressure in mm.	4.4	2.2	1.3	0.7	0.3
Luminosity. Current = 2 M. A.	17	20	25	27	35
" = 4 "	36				
" = 6 "	54	61	73	83	109
95 % Nitrogen.					
Pressure in mm.	3.60	2.75	0.8	0.45	
Luminosity. Current = 2 M. A.	18	19	27	31	
" = 6 "	56	56	80	92	
71 % Nitrogen.					
Pressure in mm.	4.11	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	13	16	19	24	
" = 6 "	40	47	55	70	
47 % Nitrogen.					
Pressure in mm.	3.95	2.35	0.8	0.38	
Luminosity. Current = 2 M. A.	8	9	13	17	
" = 6 "	24	26	38	50	
36 % Nitrogen.					
Pressure in mm.	2.75	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	7	8	10	13	
" = 6 "	19	20	25	30	
5 % Nitrogen.					
Pressure in mm.	3.32	1.75	0.75	0.35	
Luminosity. Current = 2 M. A.	Invisible	Barely	1	2	
" = 6 "		visible	2	5	

TABLE I.—(*Continued.*)

NITROGEN BAND $\lambda = 6542$.

Pure Nitrogen.					
Pressure in mm.	4.4	2.2	1.3	0.7	0.3
Luminosity. Current = 2 M. A.	26	29	35	45	51
" = 6 "	78	88	108	133	158
91 % Nitrogen.					
Pressure in mm.	3.60	2.75	0.8	0.45	
Luminosity. Current = 2 M. A.	27	28	42	47	
" = 6 "	80	84	125	140	
71 % Nitrogen.					
Pressure in mm.	4.11	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	15	22	30	35	
" = 6 "	45	65	92	105	
47 % Nitrogen.					
Pressure in mm.	3.95	2.35	0.8	0.38	
Luminosity. Current = 2 M. A.	13	14	19	23	
" = 6 "	40	42	58	70	
36 % Nitrogen.					
Pressure in mm.	2.75	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	10	11	14	17	
" = 6 "	28	30	40	48	
5 % Nitrogen.					
Pressure in mm.	3.32	1.75	0.75	0.35	
Luminosity. Current = 2 M. A.	Invisible	1	2	3	
" = 6 "		1	4	6	

NITROGEN BAND $\lambda = 6622$.

Pure Nitrogen.					
Pressure in mm.	4.4	2.2	1.3	0.7	0.3
Luminosity. Current = 2 M. A.	47	48	56	65	83
" = 6 "	145	148	168	193	257
91 % Nitrogen.					
Pressure in mm.	3.60	2.75	0.8	0.45	
Luminosity. Current = 2 M. A.	45	46	55	68	
" = 6 "	135	140	168	205	
71 % Nitrogen.					
Pressure in mm.	4.11	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	33	36	44	59	
" = 6 "	100	105	134	175	
47 % Nitrogen.					
Pressure in mm.	3.95	2.35	0.8	0.38	
Luminosity. Current = 2 M. A.	21	23	31	38	
" = 6 "	62	68	95	115	
36 % Nitrogen.					
Pressure in mm.	2.75	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	16	18	23	27	
" = 6 "	45	50	65	75	
5 % Nitrogen.					
Pressure in mm.	3.32	1.75	0.75	0.35	
Luminosity. Current = 2 M. A.	3	4	7	7	
" = 6 "	7	10	16	17	

TABLE I.—(*Concluded.*)

NITROGEN BAND $\lambda = 6701$.

Pure Nitrogen.					
Pressure in mm.	4.4	2.2	1.3	0.7	0.3
Luminosity. Current = 2 M. A.	44	49	53	61	68
" = 6 "	133	145	160	178	215
91 % Nitrogen.					
Pressure in mm.	3.60	2.75	0.8	0.45	
Luminosity. Current = 2 M. A.	41	42	53	59	
" = 6 "	122	125	160	180	
71 % Nitrogen.					
Pressure in mm.	4.11	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	32	35	43	48	
" = 6 "	95	105	120	145	
47 % Nitrogen.					
Pressure in mm.	3.95	2.35	0.8	0.38	
Luminosity. Current = 2 M. A.	20	22	27	33	
" = 6 "	60	65	80	100	
36 % Nitrogen.					
Pressure in mm.	2.75	1.73	0.8	0.35	
Luminosity. Current = 2 M. A.	13	16	21	23	
" = 6 "	35	43	58	64	
5 % Nitrogen.					
Pressure in mm.	3.32	1.75	0.75	0.35	
Luminosity. Current = 2 M. A.	2	3	5	5	
" = 6 "	2	7	12	13	

In Table I. are given observations on the luminosity of the hydrogen line $\lambda = 6563$, and nitrogen bands $\lambda = 6465$, 6542, 6622 and 6701, when current, pressure of gas and proportion of the gases in the mixture are changed. These values are represented graphically for the hydrogen line $\lambda = 6563$ and nitrogen bands $\lambda = 6622$ and 6701 in Figs. 1, 2 and 3, respectively.

From an examination of these tables and curves we conclude that

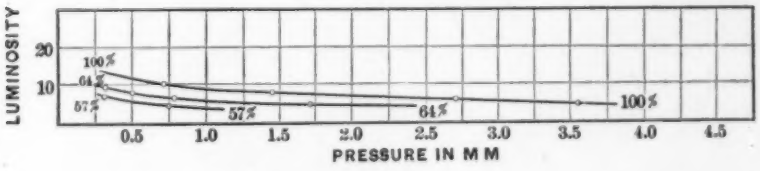

Fig. 1.

Diagram showing the relation between luminosity of hydrogan line $\lambda = 6563$ with pressure, when nitrogen is added in various proportions. Current is constant and equal to 2.0 M.A. The proportion of hydrogen present is marked in per cents.

when a gas is present in a mixture in large proportion, the luminosity of a line in its spectum varies directly with the current when the pressure of the gas is constant; and also that the luminosity varies in an inverse ratio with the pressure when the current is constant. In other words, when a gas is present in a mixture in large proportion, the luminosity of its spectrum follows the same laws with respect to changes of current and gas pressure that it does when the gas is pure. The observations made on the luminosity of the gas present in but small proportion, point to departures from these laws. With pressure constant, the luminosity then increases more slowly than the current. And with current constant, the luminosity increases at first with decrease of pressure; but after the pressure has been diminished to a certain amount, the luminosity remains nearly constant for any further decrease of pressure.

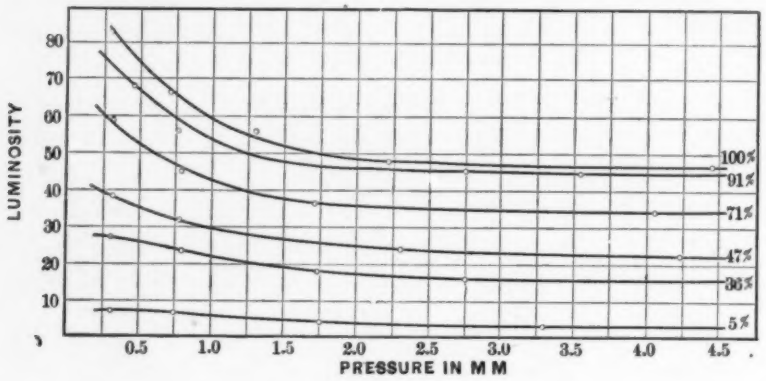

Fig. 2.

Diagram showing the relation between luminosiiy of nitrogen band $\lambda = 6622$ with pressure, when hydrogen is added in various proportions. Current is constant and equal to 2.0 M. A.

In order to further test this result, observations were made on a mixture in which one element was reduced to a very small amount. The five per cent. hydrogen-nitrogen mixture was pumped out of the apparatus till the manometer showed a pressure of 0.5 mm. of mercury. Pure hydrogen was introduced till pressure became 3.8 mm. The nitrogen bands were still visible, though faint. But now, in order to render the standard spectrum of the same luminosity as the gas spectrum, the slit through which passed the light from the

TABLE II.

Luminosity of nitrogen bands in a gas mixture in which nitrogen is present in very small amount.

$\lambda = 6622$.			
Pressure of gas in mm.	3.8	2.7	0.7
Luminosity. Current = 2 M. A.	5	6	6
" = 4 "	9	11	12
" = 6 "	11	14	15
$\lambda = 6701$.			
Luminosity. Current = 2 M. A.	4	5	6
" = 4 "	7	9	11
" = 6 "	9	11	13

standard lamp had to be so much reduced in width that the error of observation became considerable. Consequently the intensity of the standard light was reduced. After this, readings with a fair degree of accuracy were possible. Table II. gives the observations obtained for this mixture of hydrogen and nitrogen, at different pressures.

Table II. makes it appear certain that when a gas is present in a mixture in very small amount, the luminosity of its spectral lines does not vary as the exciting current, but that with increase of current the luminosity increases more slowly than the current. This shows us that with a mixture of gases at constant pressure, it is possible to find lines of the two gases that for a certain current strength will

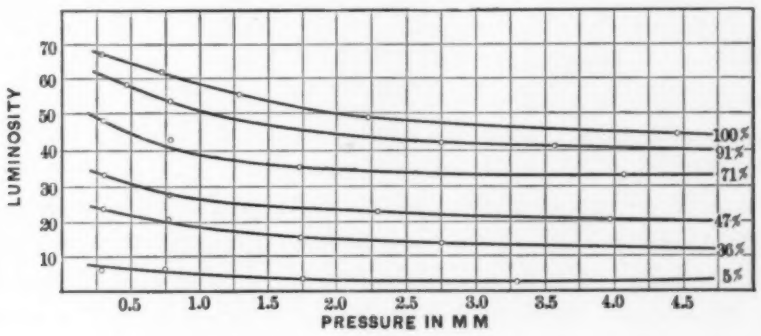

Fig. 3.

Diagram showing the relation between luminosity of nitrogen band $\lambda = 6701$ with pressure when hydrogen is added in various proportions. Current is constant and equal to 2.0 M. A.

be of equal luminosity ; while for a smaller current, the line belonging to the element present in smaller proportion will be stronger than the line belonging to the element present in larger proportion, whereas, for a greater current it will be weaker. We therefore conclude that the relative luminosity of lines of different elements present in a mixture is a function of the proportion of the elements in the mixture, the strength of the current rendering the gas luminous, and the pressure of the gas.

The curves plotted in Figs. 1, 2, and 3 show that the luminosity of the spectrum of a gas increases with an increase in the proportion of that constituent of the mixture ; but to render more evident the precise relation between luminosity and percentage of a constituent

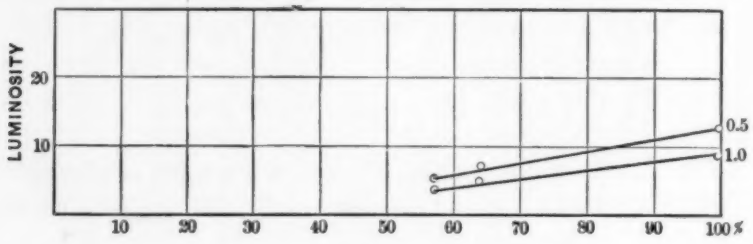

Fig. 4.

Diagram showing the relation between luminosity of hydrogen line $\lambda = 6565$ and amount of hydrogen present. Current is constant and eqcal to 2.0 M. A. Curves are for pressure of 0.5 and 1.0 mm. of mercury.

in a mixture, curves are constructed in which these two quantities are the coördinates, and pressure and current are constant. Such curves for different constant pressures, and for a uniform current strength of two milliampères are given for hydrogen line $\lambda = 6563$ and nitrogen bands $\lambda = 6622$ and 6701 in Figs. 4, 5, and 6. These curves show that the luminosity of a spectral line of an element is nearly but not quite proportional to the percentage of that element present in the mixture. All of the curves are convex toward the coördinate axis on which the percentage composition is measured. From the tables it is also seen that as the percentage of one element of the mixture is increased, the spectral lines of that element become brighter while the lines of the other component become fainter. It will be noticed that if these curves be produced to the

limit of zero amount of the component in the mixture, the luminosity will still have a positive value. This is probably an error of observation due to the tendency to make the comparison spectrum too bright when balancing its luminosity against that of a feeble line.

During the course of these experiments the bands of the so-called "second hydrogen spectrum" were often visible and whenever sufficiently bright their luminosity was measured. The observations thus obtained seem to show that the luminosity of these bands depends simply upon the pressure of the gas and strength of current and is independent of the amount of hydrogen in the mixture. The data thus far obtained are probably insufficient to form any definite conclusion; but if future experiments should verify these results, they would have an important bearing upon the still unsettled question of the origin of these bands.

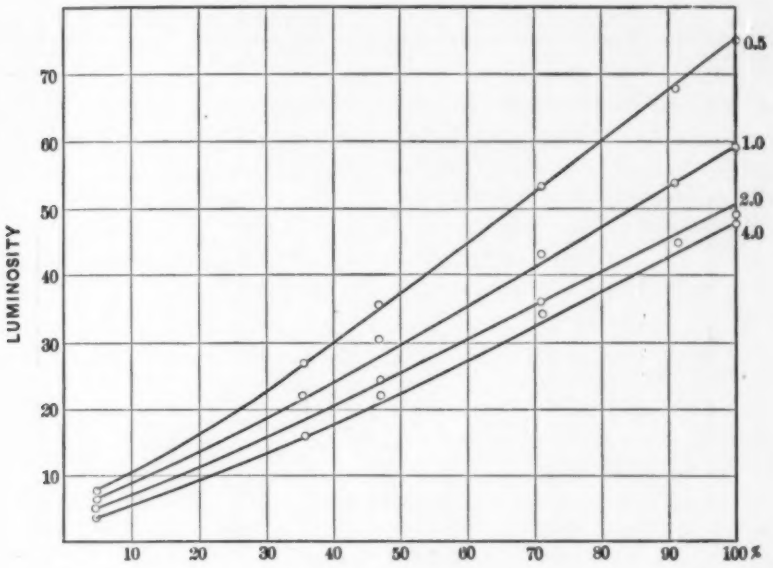

Fig. 5.

Diagram showing the relation between luminosity of nitrogen band $\lambda = 6622$ and amount of nitrogen present. Current is constant and eqaal to 2.0 M. A. Curves are for pressures of 0.5, 1.0, 2.0 and 4.0 mm. of mercury.

It was also noticed that when nothing was placed between the pump and discharge tube to prevent mercury vapor entering the

tube, the luminosity of the mercury lines was independent of the pressure of the gas mixture as determined by the MacLeod gauge. This simply means that the tension of the mercury vapor was constant and independent of the pressure of the other gas contained in the tube.

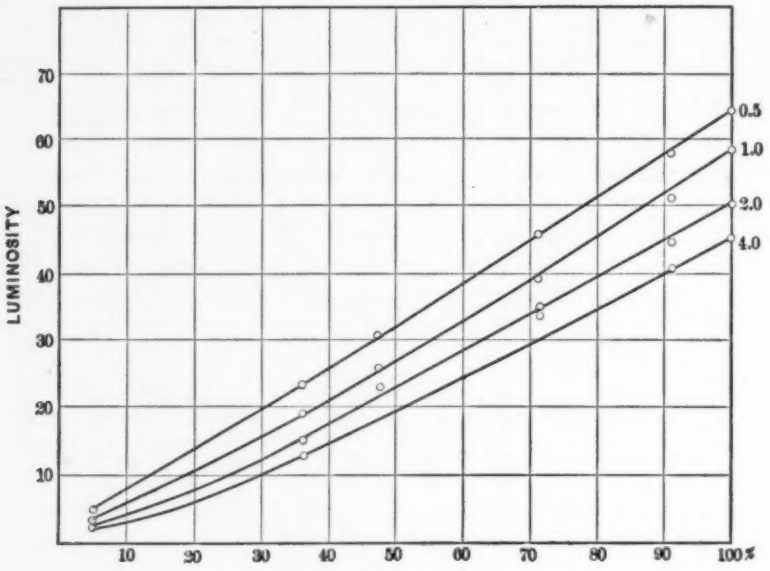

Fig. 6.

Diagram showing the relation between luminosity of nitrogen band $\lambda = 6701$ and amount of nitrogen present. Current is constant and equal to 2.0 M. A. Curves are for pressures of 0.5, 1.0, 2.0 and 4.0 mm. of mercury.

NEW BOOKS.

A Treatise on Magnetism and Electricity. By ANDREW GRAY, F.R.S. In two volumes. 8vo. Vol. I., pp. 479. London, Macmillan & Company. 1898.

It is natural that Professor Gray should have become strongly impressed during the preparation of his three volumes on *Absolute Measurements*, with the desirability of a rediscussion of the whole subject of electricity and magnetism from the point of view of action in a medium, and it is very greatly to the advantage of every student of electricity that Professor Gray has undertaken to "put together a statement which from the beginning should regard electric and magnetic forces as existing in a space-pervading medium in which the electric and magnetic energies are stored, and by which they are handed on from one place to another with a finite velocity of propagation."

The first volume of this "statement" indicates very distinctly that we are to have a treatise passing all its forerunners in that most important quality, the bringing together of fact and theory, which quality is, in fact, a marked feature of the author's *Absolute Measurements*.

It is gratifying to one who has been helped by the writings of Heaviside to read Professor Gray's frank expression of his great obligations to this writer. "No writer," says Professor Gray, "on Maxwell's theory has done so much to elucidate and render consistent its various parts, or contributed so much to the practical solution of problems of wave propagation. Of the importance of Mr. Heaviside's views on general theory I am strongly convinced, and no adequate presentment of Maxwell's theory can be obtained in which they are not largely adopted. Thus Volume II., which will deal more with general discussions, will be affected to a still greater degree by the results of Mr. Heaviside's work." If any one wishes an example of Mr. Heaviside's work let him read his overreaching discussion of Helmholtz's theory of dispersion published about three years ago in the *London Electrician*.

Professor Gray's *Treatise* is quite sufficiently characterized in his own words quoted above. The most striking features of the book are the chapter on the Terrestrial Magnetism, the chapter on the Magnetism of Iron Ships and the compensation of the Compass, the chapter on general Dynamical Theory, and the chapter on Fluid Motion. It seems to the writer that we have here for the first time in a Treatise on Electricity and Magnetism adequate discussions of these latter subjects.

W. S. FRANKLIN.

An Elementary Course of Infinitesimal Calculus, by HORACE LAMB, M.A., F.R.S., Professor of Mathematics in the Owens College, Victoria University, Manchester; formerly Fellow of Trinity College, Cambridge. Pp. xx + 616 pages. Cambridge University Press. 1897.

The number of elementary text-books on the infinitesimal calculus is increasing rapidly. The increase is due mainly to a desire on the part of authors to supply the needs of students in the scientific and technical colleges. There evidently is a wide difference of opinion as to what should constitute a course in calculus for these students and for beginners in general. Some wish to confine the work of the teacher to the narrowest possible limits, and think it is his duty merely to so plan his class-work that his pupils shall become, at a minimum expenditure of thought, able to perform the simplest operations of the calculus. It is a fact that students can be trained to use the calculus in a fairly trustworthy way, while all the time they regard it with the same unthinking innocence with which a child looks upon a penny-in-the-slot machine. There are also teachers who are wanting in sympathy with the difficulties of beginners and who scorn the needs of technical men. They delight in unnecessarily hard proofs, dwell long on points which should scarcely be referred to in an elementary course, and give thanks for anything in the subject that seems unadapted for practical use. The reviewer is inclined to think that all beginners require the *same* kind of a text-book in calculus. The cry of technical men for an introductory course especially suited to their needs is, to a certain extent, an evidence of the failure of the average text-book to meet properly the requirements of a beginner.

The two leading features of Professor Lamb's volume are the excellent exposition of the fundamental principles of the subject and the abundance of interesting practical applications. The excellence of the exposition is manifest both in the lucidity and attractiveness of the style and in the elegance and simplicity, combined with rigor, which, in general, characterize the deduction of the theorems. The principles of the calculus are explained in a clear and simple manner, but simplicity is not gained at the expense of sound logic. The chief merit of the work is this: The subject is presented in such a way that the young student can readily comprehend it, while at the same time the inherent difficulties of its fundamental conceptions are pointed out and he is shown the modern methods of meeting these difficulties. The marked differences, both in arrangement and subject matter, between this book and the conventional text-book are evidences of the author's good understanding of and sympathy with the needs and aims of students both in the general and in the technical courses in the colleges. Throughout the book stress appears to be laid upon a maxim which is too often neglected in mathematics and allied subjects. This maxim is that one should be able to translate

the mathematical symbols and expressions which he happens to be using into plain English. The free use of graphical methods in this work adds to the interest and clearness of the exposition and the geometrical representation and interpretation of principles and results is much more common than in most books on the subject. One instance of many which will be new to readers of text-books in English is Professor Klein's geometrical representation of a particular expansion in Maclaurin's series (p. 572). The author also carefully reminds the student of the limitations to the use of graphical methods (p. 21). The book contains a great variety of exercises of a practical kind; the problems cover a wide range, and many branches of applied mathematics have been drawn upon for contributions. Some of the examples are concrete, and their solution requires exact numerical computation. This is a commendable feature which will be helpful to many; for interest is aroused and confidence is strengthened by arithmetical numbers. Moreover, practice in the art of computation will be of great benefit to some pupils. Hyperbolic functions are used. Many interesting and valuable historical and incidental notes are scattered through the book.

In the first chapter, the fundamental notions of continuity and of limiting values are explained. Part of this important chapter may be omitted in the first reading, or it may be studied along with the subsequent portions of the work. The general formulæ of differentiation and the formulæ for some particular functions are deduced in Chapter II. A short table of the principal formulæ would be found convenient. Applications of the first derivative to the theory of equations, to maxima and minima and to geometry are then introduced. Chapter IV. treats of derivatives of higher order than the first. The symbol D for $\frac{d}{dx}$ is frequently employed. The larger part of the chapter is taken up with the second derivative and its interpretations and applications. By means of geometry and the theorem of mean value, as much of Taylor's expansion is obtained as is ordinarily required in geometry and dynamics. In addition to the ordinary use of the second derivative in determining the convexity and concavity of a curve and in finding the maximum and minimum values of a function, there is an interesting application to the error involved in the process of interpolating by proportional parts.

The following three chapters are devoted to integration. Chapter V. includes matter which, in other books, is usually spread over several chapters. The problem of integration is defined as the inverse of differentiation, and the fundamental formulæ of integration are obtained. Integration by parts, by changing the variable, and by successive integrations are discussed; partial fractions and irrational and trigonometric functions are treated. In the next chapter, the meaning and properties

of definite integrals are set forth very clearly. A definite integral is defined as the limiting value of a sum of a certain type and this sum is shown to be convergent in a particular case which is sufficient for all practical purposes. The geometrical applications given in Chapter VII. include the usual determinations of plane areas, volumes, lengths of curves, and surfaces in the order mentioned. Attention is drawn to the sign which must be attributed to an area, and reference is made to the generalized sense of the formulæ for areas of curves. The article on the theory of Amsler's planimeter and rectification of curves may be criticised as being rather brief. It is also unfortunate that there is not a fuller treatment of multiple integration. The value of the chapter is increased by interesting articles on approximate integration and mean values.

The seven chapters described above cover 287 pages. This portion of the book, together with something on curves from Chapter IX., a few articles on curvature and envelopes from Chapter X., and some sections on differential equations from Chapters XI., XII., will provide a good first course for one entering upon the study of the calculus. This course will supply students of elementary physics and engineering with the mathematics ordinarily necessary in their work. Moreover, the treatment and exposition of the subject have been of such a character, that pupils can hardly fail to receive a clear understanding of the meaning, the method, and the spirit of the calculus. It should be a comparatively easy matter for them to make any special simple applications involving this kind of mathematics. The remaining sections of the volume, for the most part, deal with subjects of a more special character which are of particular interest either to physicists, engineers, or those studying pure mathematics. The larger number of the problems are taken from branches of physics and engineering, and some of them will require considerable explanation before they can be understood by a beginner. Many of these examples, however, can be used easily and with advantage in classes which have not received instruction in the branches of study from which the exercises have been drawn. The indication of so many special applications of the calculus will widen the horizon of both teacher and taught, and will provide ample means by which the interest of pupils in the study can be stimulated and their understanding of the subject strengthened.

Chapter VIII. deals with physical applications. Mean densities and mass-centers are found in the cases of line distribution, plane areas, surfaces, and solids. There are also articles devoted to mean pressure, mean square of distance, inertia, radius of gyration, and applications to distribution stresses and homogeneous strains. Special curves are discussed in the next sixty pages. Remarks on the origin and use of some of the curves enliven the articles. Curvature is the subject of the sixty

pages of Chapter X., in which the usual matter on radius of curvature, envelopes, evolutes and involutes is presented. Its latter half consists of sections which are especially applicable to kinematics. Such, for instance, are those on center of rotation and instantaneous center. The determination of the proper shapes to be given to the teeth of wheels is touched upon. Within the compass of the one hundred pages forming Chapters XI., XII., there are given the ordinary differential equations of the first and second orders which one is likely to meet in elementary work. A little of the general theory is explained and many practical examples are provided. The author avoids imaginary roots in the auxiliary equation of the linear equation, but the method is necessarily cumbersome and may appear somewhat arbitrary to the student. The last two chapters of the volume are necessary for any one who wishes to have a thorough grounding in the subject. In Chapter XIII. is discussed the integration and differentiation of power-series. The exposition of this fundamental, but too frequently neglected, matter is all that can be desired. It ends with articles on the integration of differential equations by series, and on expansions by means of differential equations. Taylor's theorem and its applications are considered in Chapter XIV. The proof of Maclaurin's and Taylor's theorems are elegant and rigorous. Applications are made to the order of contact of curves, to maxima and minima, and to the infinitesimal geometry of plane curves. A rigid proof of the commutative property in partial derivatives of orders higher than the first is given, and the extended form of Taylor's theorem is deduced. The special application of the latter to maxima and minima is illustrated geometrically, but there is no discussion of the case in which it is necessary to consider derivatives of orders higher than the second. The appendix contains short tables of square roots, reciprocals, logarithms to bases 10 and e, and circular and hyperbolic functions, and the book is provided with an index. The theory of indeterminate forms and the determination of asymptotes and singularities of curves are omitted; the latter because the subject belongs to analytic geometry; the former because the discussion is tedious and better methods of evaluation are provided outside the calculus.

Mathematicians, physicists, and engineers are indebted to Professor Lamb for his valuable contribution to the solution of the problem: How may the infinitesimal calculus be best presented in an elementary course? His work, which is written simply, clearly, and in accordance with the standards of rigor required in modern mathematics, should do much towards improving the elementary instruction in calculus throughout the English-speaking world.

D. A. MURRAY.

CORNELL UNIVERSITY.

The Storage Battery. AUGUSTUS TREADWELL, JR., E.E. 12mo. Pp. xix + 252. New York, The Macmillan Co. 1898.

Here is a timely work that should be of genuine assistance to anyone seeking information on the storage cell. The book is apparently the result of an earnest effort on the part of the author to gather into a convenient form the principles underlying the construction, theory and operation of the storage battery. Indeed, the work is styled "A Practical Treatise" on these very things, and it will probably be found to justify its subordinate title. The book-market boasts so many "Practical Treatises" of uncertain merit that it is with some misgivings that one opens the covers of a work bearing that inscription. It is a pleasure to note that in this little compendium the author has adopted a style befitting his subject and, what is more important, has placed before the reader the opinions and results of those who have done the best work in this field of research.

A brief introduction, in which something of the history of the storage cell is entered into, is followed by five chapters devoted to descriptions of the various types of lead and bi-metallic cells. The author is very systematic in his classification into types, and there are copious references to patent records. On the whole, this position of the work makes a valuable source for reference, but is too replete with minutæ to be altogether interesting reading.

Chapter VI. is devoted to discussion of the theory of the actions which take place in the storage battery and contains much of physical and technical interest. The principal results obtained by Gladstone and Tribe, Frankland, Ayrton, Griscom and many others prominent in storage battery research are touched upon. The chapter is the most interesting in the book to the man of more purely scientific pursuits. Some will feel, I think, that the value of this chapter would be enhanced if the author had quoted less freely, giving instead the same matter in a style more connected and showing greater assimilation.

Chapters VII. and VIII. treat of the applications of the storage battery. The author properly emphasizes the great utility of accumulators in station practice, but despairs, with perhaps equal propriety, of any extended use of them for traction purposes. In the concluding chapter of the work one finds much useful information pertaining to the care of cells, and the prevention of "sulphating," "bucklings" and other diseases to which the storage battery is susceptible.

The book concludes with an appendix containing directions for the measurement of such quantities as are necessary to the study of a cell, and a table classifying batteries according to their trade names with data on capacity, efficiency and so on.

There are 100 or more illustrations distributed throughout a text which is typographically excellent. The author's style is in the main clear and

straightforward. There is an occasional lapse in English, even in so rudimental a matter as agreement between noun and verb.

C. P. MATTHEWS.

PURDUE UNIVERSITY, LAFAYETTE, IND.

La Photographie et l'étude des Nuages. Par JACQUES BOYER. Pp. 79, 21 figures. Charles Mendel, Paris. 1898.

This little book is designed to give a popular account of the methods of cloud photography and measurement, and not as an aid to the practical meteorologist, who will require more detail as to apparatus and methods than are furnished by M. Boyer. The same is true of the description of cloud forms and the historical notice which forms an introduction to the book. The general reader, however, will find an interesting, as well as correct and concise account of a branch of meteorological work whose utility is becoming more evident year by year.

The apparatus and methods described by M. Boyer are those employed by the Meteorological Bureau of France, at the observatory recently built upon the plateau des Trappes, 29 kilometers southwest of Paris. In essentials, the scheme of observation is similar to that employed at Blue Hill, Mass., Kew, Upsala, etc. Two theodolites, whose telescopes are replaced by cameras of 255 millimeters focal length, are located 1318 meters apart; and by aligning them toward a given cloud, its distance is deduced from a comparison of the positions of the images upon the two photographic plates, in conjunction with the circle readings of the theodolites. This comparison is rendered precise by the use of a micrometer of long focus and wide field. The scheme of reducing the observations is not given in detail.

M. Boyer also enumerates five methods of obtaining satisfactory negatives of difficult cirrus cloud forms by the use of glass and fluid screens, and by polarized light, all of which are familiar to specialists in this work. The specimens of cloud-photographs given in the book are satisfactory, and figure 20 is of especial interest as showing the cloud formation above a destructive tornado.

E. T. TURNER.

Leçons de Chimie Générale. By P. SCHÜTZENBERGER. Published by O. BOUDOUARD. Pp. 586. Paris, Octave Doin. 1898.

This is a posthumous work of the late Professor Schützenberger of the Collège de France, published by his assistant and pupil, M. Boudouard. It consists of the substance of Schützenberger's lectures and forms a supplement to the author's well-known compendium, the *Traité de Chimie Générale.*

While addressed to chemists, or more particularly to physical chemists,

there is much in this volume which will interest the physicist. Indeed the chapters on the properties of gases and liquids, on solutions, on cryoscopy, and on boiling-point methods, form an important portion of the book. The last-named chapter discusses at considerable length the results obtained by Raoult, Jones, Beckmann and others.

While the chief function of these lectures seems to have been to impress upon chemists the importance of utilizing such relations as that between the boiling point, the refracting power, etc., and the molecular composition of bodies, and of establishing the position of the spectroscope and the spectrophotometer as instruments in chemical research, it will, on the other hand, greatly interest the physicist and will serve to remind him of the ever-increasing extent to which the sciences of physics and chemistry are encroaching each upon the field of the other and are becoming parts of the same science. E. L. N.

Lecture-Notes on the Theory of Electrical Measurements. By WILLIAM A. ANTHONY. Pp. vi+90. New York, John Wiley & Sons. 1898.

This is really a syllabus of Professor Anthony's course of lectures at the Night School of the Cooper Union. Many topics are merely indicated, as at the opening of the chapter on the Magnetic Field, where we find the following list: "*Definition of a Magnet; Natural and Artificial Magnets; Bar Magnets, Horseshoe Magnets; Distribution of Magnetic Force; Ends where Force is Manifested called Poles; The two Poles are not alike.*" This constitutes simply a list of memoranda, which can be useful only in connection with the lectures themselves, or with some text-book in which more extended treatment is given. In addition to these memoranda, however, we find definitions of many important points.

The first section of the book, which is devoted to a series of brief statements characterizing the c. g. s. system of units, contains definitions of force, work, energy, etc. The relation of the c. g. s. system to the so-called practical units to the gravitation units is clearly and definitely pointed out. The treatment is quite as rigorous and exact as could be expected when we consider the class of students for whom it is intended. Mathematical symbols are used for the most part merely for the purpose of indicating the relationships between physical quantities, such as those which exist in Ohm's law, etc. Occasionally the development of a formula is undertaken.

At the end of each section are problems of a practical character. The method in which the book is to be used is indicated in the preface, where the student is advised of the great importance, as soon as possible after each lecture, of going over the ground and filling in his notes, thereby clearing up subjects that may have appeared obscure and fixing his knowledge of the topics treated.

Without some such work this volume would largely fail of its purpose. As an adjunct, however, to a thorough course of lectures with experimental demonstrations, it affords in brief and compact form a summary of those things which it is most important for the student to remember.

E. L. N.

Ostwald's Klassiker der Exakten Wissenschaften. No. 93, *Drei Abhandlungen über Kartenprojection.* VON LEONHARD EULER. Pp. 77. No. 96, SIR ISAAC NEWTON'S *Optik oder Abhandlung über Spiegelungen, Brechungen, Beugungen und Farben des Lichts.* Pp. 132. Leipzig, Wilhelm Engelmann. 1898.

Number 93 of Ostwald's series of scientific reprints contains three papers by Euler on the theory of map drawing. The original papers which appeared in the Proceedings of the St. Petersburg Academy in 1777 are not easily accessible. On this account and for the privilege of reading the memoirs in German rather than in the original Latin, many of those who are interested in this subject will be grateful.

Number 96 of the same series contains a German translation of Newton's Optics, together with the reproduction of an old portrait which represents Sir Isaac in his eighty-fourth year. The volume, which is translated by William Abendroth, of Dresden, contains only Book I of Newton's treatise.

E. L. N.

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